

Limits & Continuity

1.1 – Limits by Tables

Limits

The word “limit” is used in everyday life to describe the ultimate behavior of something, as in the “limit of one’s endurance” or the “limit of one’s patience.” In mathematics, the word “limit” has a similar but more precise meaning.

The notation $x \rightarrow 2$ (read “x approaches 2”) means that x takes values arbitrarily close to 2 without ever reaching 2.

For example, let’s approach 2 from the left.

Let’s approach 2 from the right.

Please note that $x \rightarrow 2$ means that x takes values closer and closer to 2 *but never equals 2*.

The limit of a function may or may not exist. If the limit of a function does exist, it must be a *single number*.

Translate into words:

$x \rightarrow 2^-$ x approaches 2 from the left

$x \rightarrow -2$ x approaches -2 (from both sides)

$x \rightarrow -2^+$ x approaches -2 from the right

One-Sided Limits

In a one-sided limit, the variable approaches the number from one side only. For example, the limit as x approaches 3 from the left, denoted $x \rightarrow 3^-$, means the limit using only x-values to the left of 3, such as 2.9, 2.99, 2.999.... The limit as x approaches 3 from the right, denoted $x \rightarrow 3^+$, means the limit using only x-values to the right of 3, such as 3.1, 3.01, 3.001....

Left and Right Limits

$\lim_{x \rightarrow c^-} f(x)$ = the limit of $f(x)$ as $x \rightarrow c$ from the left.

$\lim_{x \rightarrow c^+} f(x)$ = the limit of $f(x)$ as $x \rightarrow c$ from the right.

Two-sided, Left and Right Limits

$\lim_{x \rightarrow c} f(x) = L$ iff both one-sided limits $\lim_{x \rightarrow c^-} f(x)$ and $\lim_{x \rightarrow c^+} f(x)$ exist and equal the same number L .

Limit of a Function (aka Two Sided Limit)

$\lim_{x \rightarrow c} f(x)$ is the number that $f(x)$ approaches as x approaches c ($x \neq c$).

Finding Limits

Limits may be found from tables of values of $f(x)$ for x near c .

Limits & Continuity

1.1 – Limits by Tables

Find each limit by tables correct to three decimal places

(Use the table feature in your calculator. Type the function into $y_1 =$. "Set the table" so the independent variable is set to "ask". When you type values into your table, start at a tenth below c , then a hundredth, then a thousandth, etc. Do the same for the right side of c . Copy the table to show your work)

#1) $\lim_{x \rightarrow 3} (2x + 4) = 10$

x	2.9	2.99	2.999	2.9999
$2x + 4$	9.8	9.98	9.998	9.9998

$\lim_{x \rightarrow 3^-} f(x) = 10$

3.0001	3.001	3.01	3.1	x
10.0002	10.002	10.02	10.2	$2x + 4$

$\lim_{x \rightarrow 3^+} f(x) = 10$

#2) $\lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = 2.718$

x	-1	-0.1	-0.001	-0.0001
$(1 + x)^{\frac{1}{x}}$	2.868	2.732	2.7196	2.7181

$\lim_{x \rightarrow 0^-} f(x) = 2.718$

0.0001	0.001	0.01	0.1	x
2.7181	2.7169	2.7048	2.5937	$(1 + x)^{\frac{1}{x}}$

$\lim_{x \rightarrow 0^+} f(x) = 2.718$