Limits & Continuity
1.3 – Limits By Graphing

Two-sided, Left and Right Limits

$\lim_{x \to c} f(x) = L$ iff both one-sided limits $\lim_{x \to c^-} f(x)$ and $\lim_{x \to c^+} f(x)$ exist and equal the same number L.

Ex A: For the piecewise linear function find the following limits by direct substitution.

$$f(x) = \begin{cases} x+1 & if\ x \le 3 \\ 8-2x & if\ x > 3 \end{cases}$$

a. $\lim_{x \to 3^-} f(x)$

b. $\lim_{x \to 3^+} f(x)$

c. $\lim_{x \to 3} f(x)$

Limits of Functions of Two Variables

Some limits involve two variables, with only one variable approaching a limit.

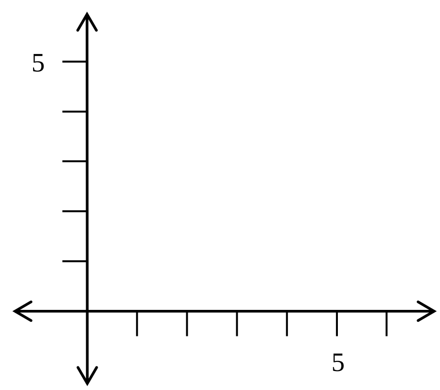
Ex B: Find the limit of each function.

#1) $\lim_{h \to 0}(x^2 + xh + h^2)$

#2) $\lim_{h \to 0}(3x^2 + 5xh + 1)$

Ex C: Finding One-Sided Limits
For the piecewise linear function find the following limits by graphing.

$$f(x) = \begin{cases} x+1 & if\ x \le 3 \\ 8-2x & if\ x > 3 \end{cases}$$



a. $\lim_{x \to 3^-} f(x)$

b. $\lim_{x \to 3^+} f(x)$

c. $\lim_{x \to 3} f(x)$

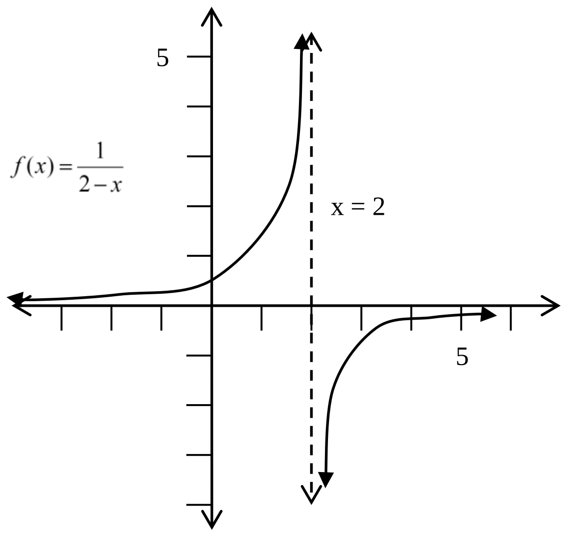
1.3 – Limits By Graphing

Infinite Limits

We may use the symbols ∞ (infinity) and -∞ (negative infinity) to indicate that the values of a function become arbitrarily large positive or arbitrarily large negative. Dashed lines on a graph, where a function approaches ∞ or -∞, are called vertical asymptotes.

Ex D: For each function graphed below, use the limit notation with ∞ and -∞ to describe its behavior as *x* approaches the vertical asymptote from the left, from the right, and from both sides.

a.



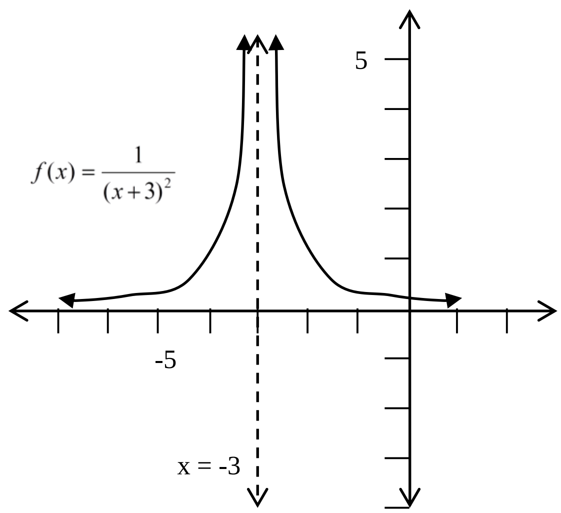
From the left

From the right

From both sides

Careful. To say that a limit exists is to say that it is a single number. Since ∞ is not a number, if $\lim_{x \to c} f(x) = \infty$, then the limit does not exists (d.n.e.)

b.



From the left

From the right

From both sides