Limits & Continuity
1.3 – Limits By Graphing

Two-sided, Left and Right Limits

$\lim_{x\to c} f(x) = L$ iff both one-sided limits $\lim_{x\to c^-} f(x)$ and $\lim_{x\to c^+} f(x)$ exist and equal the same number L.

Ex A: For the piecewise linear function find the following limits by direct substitution.

$$f(x) = \begin{cases} x+1 & if\ x \le 3 \\ 8-2x & if\ x > 3 \end{cases}$$

a. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(x+1) = (3)+1$
$= 4$

b. $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(8-2x) = 8-2(3)$
$= 8-6$
$= 2$

c. $\lim_{x\to 3} f(x)$ = dne

Limits of Functions of Two Variables

Some limits involve two variables, with only one variable approaching a limit.

Ex B: Find the limit of each function.

#1) $\lim_{h\to 0}(x^2 + xh + h^2) = x^2 + x(0) + (0)^2$
$= x^2$

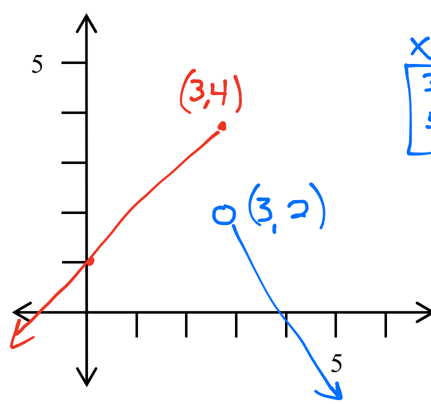
#2) $\lim_{h\to 0}(3x^2 + 5xh + 1) = 3x^2 + 5x(0) + 1$
$= 3x^2 + 1$

Ex C: Finding One-Sided Limits
For the piecewise linear function find the following limits by graphing.

$$f(x) = \begin{cases} x+1 & if\ x \le 3 \\ 8-2x & if\ x > 3 \end{cases}$$

x	x+1
3	4
0	1

x	8-2x
3	2
5	-2



a. $\lim_{x\to 3^-} f(x) = 4$

b. $\lim_{x\to 3^+} f(x) = 2$

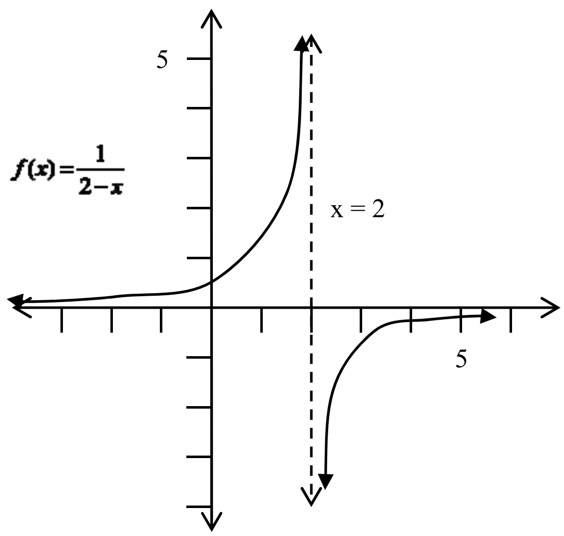
c. $\lim_{x\to 3} f(x)$ = d.n.e.

Infinite Limits

We may use the symbols ∞ (infinity) and -∞ (negative infinity) to indicate that the values of a function become arbitrarily large positive or arbitrarily large negative. Dashed lines on a graph, where a function approaches ∞ or -∞, are called vertical asymptotes.

Ex D: For each function graphed below, use the limit notation with ∞ and -∞ to describe its behavior as x approaches the vertical asymptote from the left, from the right, and from both sides.

a.



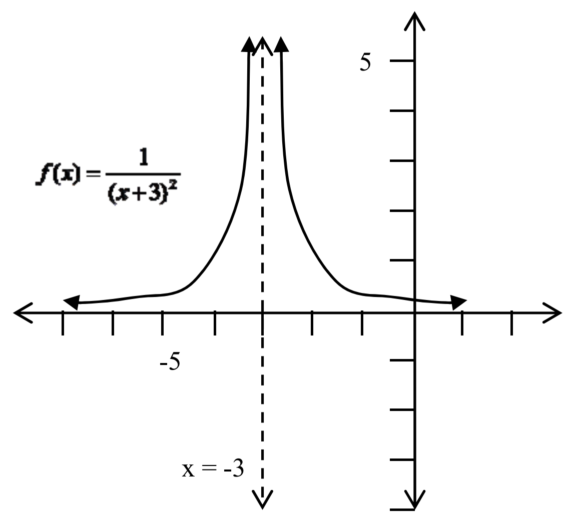
From the left $\lim_{x\to 2^-} f(x) = \infty$, dne

From the right $\lim_{x\to 2^+} f(x) = -\infty$, dne

From both sides $\lim_{x\to 2} f(x) =$ dne

Careful. To say that a limit exists is to say that it is a single number. Since ∞ is not a number, if $\lim_{x\to c} f(x) = \infty$, then the limit does not exists (d.n.e.)

b.



From the left $\lim_{x\to 3^-} f(x) = \infty$, dne

From the right $\lim_{x\to 3^+} f(x) = \infty$, dne

From both sides $\lim_{x\to 3} f(x) = \infty$, dne