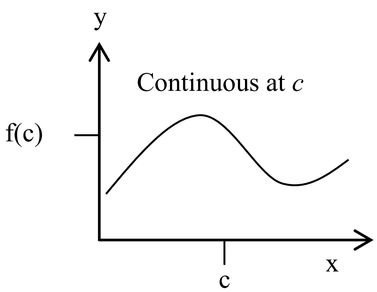
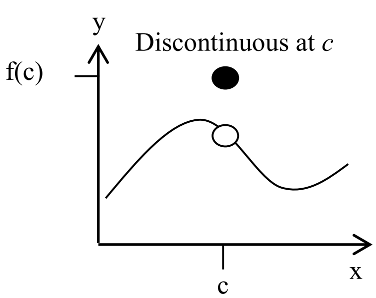
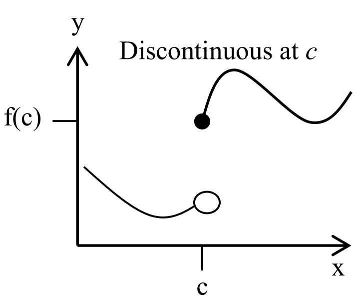
Limits & Continuity
1.4 – Continuity

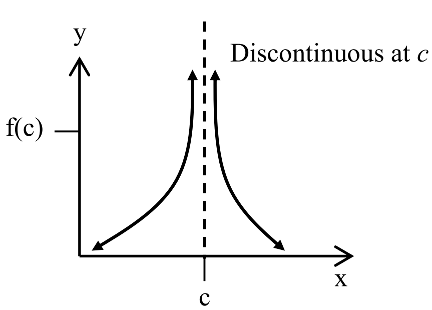
Continuity from PreCalculus

A function is said to be continuous at *c* if its graph passes through the point at $x = c$ without a "hole" or a "jump"









Continuity from Calculus

A function *f* is continuous at *c* if the following three conditions hold:

1. $f(c)$ is defined
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

f is *discontinuous* at *c* if one or more of these conditions fails to be true.

Which Functions Are Continuous?

If functions *f* and *g* are continuous at *c*, then the following are also continuous at *c*:

1. $f \pm g$
2. $a \bullet f$ [for any constant *a*]
3. $f \bullet g$
4. f/g [if $g(c) \neq 0$]
5. $f(g(x))$ [for *f* continuous at $g(c)$]

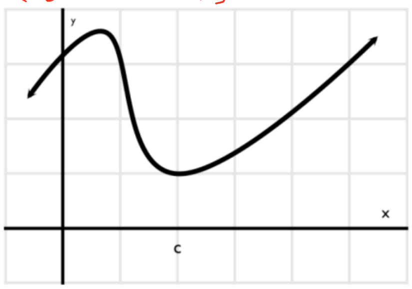
All polynomial functions are continuous. Rational functions are not continuous when the denominator = 0 (vertical asymptote). Piece-wise functions have the potential to be continuous or not.

Limits & Continuity

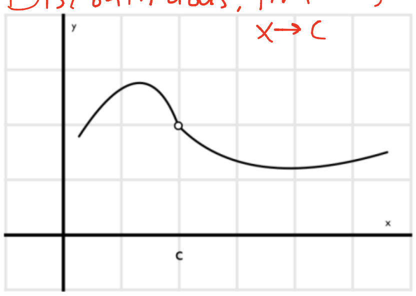
1.4 – Continuity

Ex A: Determine if each function is continuous. If discontinuous, state why.

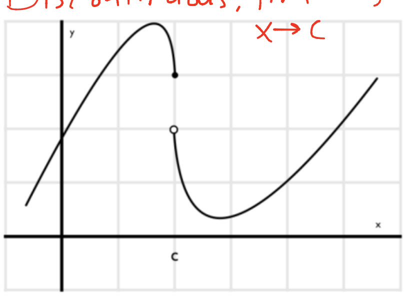
#1) Continuous



#2) Discontinuous, $\lim_{x \to c} f(x) \neq f(c)$



#3) Discontinuous, $\lim_{x \to c} f(x) = dne$.



Ex B: Determine if each function is continuous. If discontinuous, state where it is discontinuous and why.

#1) $f(x) = x^3 - 3x^2 - x + 3$

Continuous

#2) $f(x) = \frac{1}{(x-1)^2}$

VA

$(x-1)^2 = 0$

$x - 1 = 0$

$x = 1$

Discontinuous @ $x = 1$

The $\lim_{x \to 1} f(x) = dne$

#3) $f(x) = \begin{cases} 2x + 1 & if\ x < 2 \\ -2x + 9 & if\ x \geq 2 \end{cases}$

$f(x) = 2x + 1$

$f(2) = 2(2) + 1$

$= 4 + 1$

$f(2) = 5$

$f(x) = -2x + 9$

$f(2) = -2(2) + 9$

$f(2) = -4 + 9$

$f(2) = 5$

Continuous

#4) $f(x) = \begin{cases} x^2 + 1 & if\ x < 4 \\ 5x - 1 & if\ x \geq 4 \end{cases}$

$f(x) = x^2 + 1$

$f(4) = (4)^2 + 1$

$f(4) = 16 + 1$

$f(4) = 17$

$f(x) = 5x - 1$

$f(4) = 5(4) - 1$

$f(4) = 20 - 1$

$f(4) = 19$

Discontinuous @ $x = 4$

$\lim_{x \to 4} f(x) = dne$