Advanced Derivative Rules
4.2B – Products, Quotients & Trigonometry

A: Differentiate using the Product Rule

#1) $\frac{d}{dx}(6x^3\cos(x))$

$= \frac{d}{dx}(6x^3)\cdot\cos(x) + 6x^3\cdot\frac{d}{dx}(\cos(x))$

$= 18x^2\cos(x) + 6x^3(-\sin(x))$

$= 18x^2\cos(x) - 6x^3\sin(x)$

#2) If $f(x) = 4x^2\sin(x)$, find $f'(x)$.

$f'(x) = (4x^2)'\sin(x) + 4x^2\cdot[\sin(x)]'$

$= 8x\sin(x) + 4x^2\cos(x)$

#3) $\frac{d}{dx}[x\sin(x)]$

$= \frac{d}{dx}(x)\cdot\sin(x) + x\cdot\frac{d}{dx}(\sin(x))$

$= 1\cdot\sin(x) + x\cos(x)$

$= \sin(x) + x\cos(x)$

#4) If $f(x) = (x^2+1)\tan(x)$, find $f'(x)$.

$f'(x) = (x^2+1)'\tan(x) + (x^2+1)\cdot[\tan(x)]'$

$= (2x)\tan(x) + (x^2+1)\sec^2(x)$

$= 2x\tan(x) + (x^2+1)\sec^2(x)$

#5) $\frac{d}{dx}[\sin(x)\cos(x)]$

$= \frac{d}{dx}[\sin(x)]\cos(x) + \sin(x)\frac{d}{dx}[\cos(x)]$

$= \cos(x)\cos(x) + \sin(x)(-\sin(x))$

$= \cos^2(x) - \sin^2(x)$

#6) If $f(x) = 7\tan(x)$, find $f'(x)$.

$f'(x) = 7\sec^2(x)$

#7) $\frac{d}{dx}[\tan(x)\cot(x)]$

$= \frac{d}{dx}[1]$

$= 0$

#8) Prove that $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$

$\frac{d}{dx}[\csc(x)] = \frac{d}{dx}\left[\frac{1}{\sin(x)}\right]$

$= \frac{1'\sin(x) - 1\cdot[\sin(x)]'}{[\sin(x)]^2}$

$= \frac{0\cdot\sin(x) - \cos(x)}{\sin(x)\cdot\sin(x)}$

$= \frac{1}{\sin(x)}\cdot\frac{-\cos x}{\sin(x)}$

$= -\csc(x)\cot(x)$

Advanced Derivative Rules

4.2 B – Products, Quotients & Trigonometry

B: Differentiate using the Quotient Rule.

#9) $\frac{d}{dx}\left(\frac{\sin(x)}{7x-5}\right)$

$$= \frac{\frac{d}{dx}[\sin(x)]\cdot(7x-5) - \sin(x)\cdot\frac{d}{dx}(7x-5)}{(7x-5)^2}$$

$$= \frac{\cos(x)\,(7x-5) - \sin(x)\,(7)}{(7x-5)^2}$$

$$= \frac{(7x-5)\cos(x) - 7\sin(x)}{(7x-5)^2}$$

#10) $\frac{d}{dx}\left(\frac{6x^4-2x^5}{\cos(-x)}\right) = \frac{d}{dx}\left(\frac{6x^4-2x^5}{\cos(x)}\right)$

$$= \frac{(6x^4-2x^5)'\cdot\cos(x) - (6x^4-2x^5)[\cos(x)]'}{[\cos(x)]^2}$$

$$= \frac{(24x^3-10x^4)\cos(x) - (6x^4-2x^5)(-\sin(x))}{\cos^2(x)}$$

$$= \frac{(24x^3-10x^4)\cos(x) + (6x^4-2x^5)\sin(x)}{\cos^2(x)}$$

#11) $\frac{d}{dx}\left(\frac{x^2}{\sec(x)}\right) = \frac{d}{dx}\left(x^2\cos(x)\right)$

$$= (x^2)'\cos(x) + x^2[\cos(x)]'$$

$$= 2x\cos(x) + x^2(-\sin(x))$$

$$= 2x\cos(x) - x^2\sin(x)$$

#12) $\frac{d}{dx}\left(\frac{\csc(x)}{x^3-8}\right)$

$$= \frac{[\csc(x)]'(x^3-8) - \csc(x)(x^3-8)'}{(x^3-8)^2}$$

$$= \frac{-\csc(x)\cot(x)(x^3-8) - \csc(x)(3x^2)}{(x^3-8)^2}$$

$$= \frac{-(x^3-8)\csc(x)\cot(x) - 3x^2\csc(x)}{(x^3-8)^2}$$

#13) $\frac{d}{dx}\left(\frac{\cot(x)+9}{\sqrt{x}}\right)$

$$= \frac{\frac{d}{dx}[\cot(x)+9]\cdot\sqrt{x} - [\cot(x)+9]\frac{d}{dx}(x^{\frac{1}{2}})}{(x^{\frac{1}{2}})^2}$$

$$= \frac{-\csc^2(x)\cdot\sqrt{x} - [\cot(x)+9]\cdot\frac{1}{2}x^{-\frac{1}{2}}}{x}$$

$$= \frac{-\sqrt{x}\csc^2(x) - \frac{1}{2\sqrt{x}}[\cot(x)+9]}{x}$$

or

$$= \left[-x^{\frac{1}{2}}\csc^2(x) - \frac{1}{2}x^{-\frac{1}{2}}[\cot(x)+9]\right]\cdot x^{-1}$$

$$= -x^{-\frac{1}{2}}\csc^2 x - \frac{1}{2}x^{-\frac{3}{2}}[\cot(x)+9]$$

$$= -\frac{1}{\sqrt{x}}\csc^2 x - \frac{1}{2\sqrt{x^3}}[\cot(x)+9]$$

#14) $\frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)+2}\right)$

$$= \frac{\frac{d}{dx}[\sin(x)]\cdot[\cos(x)+2] - \sin(x)\cdot\frac{d}{dx}[\cos(x)+2]}{[\cos(x)+2]^2}$$

$$= \frac{\cos(x)[\cos(x)+2] - \sin(x)[-\sin(x)]}{(\cos(x)+2)^2}$$

$$= \frac{\cos^2(x) + 2\cos(x) + \sin^2(x)}{(\cos(x)+2)^2}$$

$$= \frac{2\cos(x)+1}{(\cos(x)+2)^2}$$