4.3 L'Hôpital's Rule

Calculus

Name: Solutions

Practice

Find the following. Use L'Hôpital's when possible.

1. $\lim\limits_{x \to 1} \frac{x-1}{x^2-3x+2} = \frac{0}{0}$

$\lim\limits_{x \to 1} \frac{1}{2x-3}$

$\frac{1}{2-3} = \boxed{-1}$

2. $\lim\limits_{x \to -5} \frac{x^2-2x-35}{x+5}$

$\boxed{-12}$

3. $\lim\limits_{x \to 0} \frac{4x}{\ln(x+1)} = \frac{0}{0}$

$\lim\limits_{x \to 0} \frac{4}{\frac{1}{x+1}}$

$\frac{4}{\frac{1}{1}} = \boxed{4}$

4. $\lim\limits_{x \to 0} \frac{x-1}{x^2-3x+2}$

$\boxed{-\frac{1}{2}}$

5. $\lim\limits_{x \to 1} \frac{2(x^2-1)}{\ln x^2} = \frac{0}{0}$

$\lim\limits_{x \to 1} \frac{4x}{\frac{2}{x}}$

$\frac{4}{2} = \boxed{2}$

6. $\frac{d}{dx} \frac{6x^2+x}{\sin(x)}$

$\boxed{\frac{(12x+1)\sin x - (6x^2+x)\cos x}{\sin^2 x}}$

7. $\lim\limits_{x \to 0} \frac{2x^2}{e^x-1-x} = \frac{0}{0}$

$\lim\limits_{x \to 0} \frac{4x}{e^x-1} = \frac{0}{0}$

$\lim\limits_{x \to 0} \frac{4}{e^x} = \frac{4}{e^0}$

$\boxed{4}$

8. $\lim\limits_{x \to 0} \frac{2x^2}{1-\cos(4x)}$

$\boxed{\frac{1}{4}}$

9. $\lim\limits_{x \to 0} \frac{\sqrt{4+x}-2}{x} = \frac{0}{0}$

$\lim\limits_{x \to 0} \frac{\frac{1}{2\sqrt{4+x}}}{1}$

$\frac{1}{2\sqrt{4}} = \boxed{\frac{1}{4}}$

10. $\lim\limits_{x \to -3} \frac{x-1}{x^2+7x+10}$

$\boxed{2}$

11. $\frac{d}{dx} \frac{6x^2+x}{x+1}$

$\frac{(12x+1)(x+1)-(6x^2+x)(1)}{(x+1)^2}$

$\boxed{\frac{6x^2+12x+1}{(x+1)^2}}$

12. $\lim\limits_{x \to 0} \frac{\cos x - 1}{x^2}$

$\boxed{-\frac{1}{2}}$

13. $\lim\limits_{x \to \infty} \frac{e^{2x}}{2x^2} = \frac{\infty}{\infty}$

$\lim\limits_{x \to \infty} \frac{2e^{2x}}{4x} = \frac{\infty}{\infty}$

$\lim\limits_{x \to \infty} \frac{4e^{2x}}{4} = \boxed{\infty}$

14. $\lim\limits_{x \to \infty} \frac{\ln x^2}{\ln(x+4)^3}$ Properties of Logs

$\lim\limits_{x \to \infty} \frac{2\ln x}{3\ln(x+4)}$

$\ln(\infty) = \ln(\infty+4)$

$\boxed{\frac{2}{3}}$

15. $\lim\limits_{x \to -2} \frac{x+2}{x^2+2x-3}$

$\frac{-2+2}{4-4-3} = \frac{0}{-3}$

$\boxed{0}$

16. $\frac{d}{dx}\frac{e^x}{\cos(2x)}$

$$\boxed{\frac{e^x\cos(2x)+2e^x\sin(2x)}{\cos^2(2x)}}$$

17. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-1-\frac{x}{2}}{x^2} = \frac{0}{0}$

$\lim\limits_{x\to 0}\frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}-\frac{1}{2}}{2x} = \frac{0}{0}$

$\lim\limits_{x\to 0}\frac{-\frac{1}{4}(1+x)^{-\frac{3}{2}}-\frac{1}{2}}{2} = \frac{-\frac{1}{4}-\frac{1}{2}}{2} = \boxed{-\frac{3}{8}}$

18. $\lim\limits_{x\to 10}\frac{5-\sqrt{x+15}}{x-10}$

$$\boxed{-\frac{1}{10}}$$

19. $\lim\limits_{x\to -5}\frac{x^2-2x-15}{x+5}$

$\frac{25+10-15}{0} \rightarrow$ undefined

The limit does not exist.

Test Prep: 1E, 2A, 3B, 4D, 5E, 6D

Free Response Scoring Guide

Use this only AFTER you have attempted the problem on your own.

Solutions	Points
(a) $\frac{f(0)-f(-3)}{3} = \frac{-1-5}{3} = \frac{-6}{3} = -2$ $\frac{f(3)-f(0)}{3} = \frac{-7-(-1)}{3} = \frac{-6}{3} = -2$	2: average rate of change for each interval
(b) f has an average increase over the interval $0 \le x \le 1$ and an average decrease over the interval $1 \le x \le 2$. This means somewhere over the interval $0 \le x \le 2$ there must be a value c such that $f(c)$ is a maximum. This maximum would yield $f'(c) = 0$ because f is differentiable.	1: Increasing and then decreasing 1: differentiable maximum means $f'(c) = 0$
(c) From part (a) and the MVT, there is a value r between -3 and 0 such that $f'(r) = -2$ and there is a value s between 0 and 3 such that $f'(s) = -2$. Using the MVT again, there must be a value d between r and s such that $f''(d) = 0$. (One could also argue that f decreases, increases, and decreases on the interval, therefore there are two points where $f'(x) = 0$ and between them there must be at least one point where $f''(d) = 0$.)	1: Use the MVT to state the first derivative equals -2 at some point. 1: Use the MVT again to state the second derivative equals 0 at some point.