Limits, Continuity & Derivatives
Chapter 1, 2, 3 & 4 Review I

Find the limits by using any method. Then, circle which method you used.

#1) $\lim\limits_{x \to 0} \dfrac{4}{2x - x^2} = \text{dne}$

Graphing

Table

Substitution

$x \to 0^-$ $\qquad$ $0^+ \leftarrow x$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
y	-19.05	-199	-1999	2001	201.0	21.05

$y \to -\infty$ $\qquad$ $\infty \leftarrow y$

#2) $\lim\limits_{x \to 8} \dfrac{5x^2 - 7x + 4}{x^2 - 3x} = \dfrac{5(8)^2 - 7(8) + 4}{(8)^2 - 3(8)}$

Graphing

Table

Substitution

$= \dfrac{5(64) - 56 + 4}{64 - 24}$

$= \dfrac{320 - 52}{40}$

$= \dfrac{268}{40}$

$\lim\limits_{x \to 8} \dfrac{5x^2 - 7x + 4}{x^2 - 3x} = \dfrac{67}{10}$

#3) $\lim\limits_{x \to 0} \dfrac{x^2 - 4x + 9x^3}{x} = \lim\limits_{x \to 0} \dfrac{x(x - 4 + 9x^2)}{x}$

Graphing

Table

Substitution

$= \lim\limits_{x \to 0} (x - 4 + 9x^2)$

$= (0) - 4 + 9(0)^2$

$\lim\limits_{x \to 0} \dfrac{x^2 - 4x + 9x^3}{x} = -4$

#4) $\lim\limits_{x \to 10} \dfrac{x^2 - 5x - 50}{x^2 - 100} = \lim\limits_{x \to 10} \dfrac{(x-10)(x+5)}{(x-10)(x+10)}$

Graphing

Table

Substitution

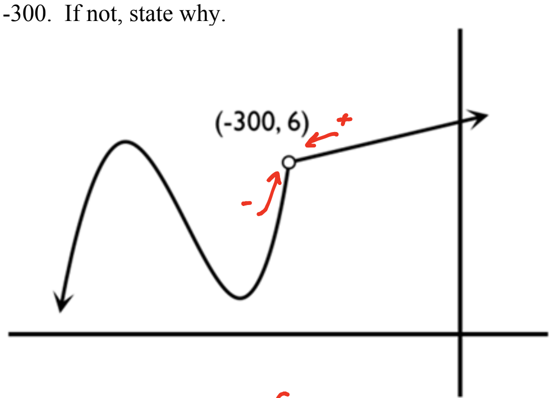
$= \lim\limits_{x \to 10} \dfrac{x+5}{x+10}$

$= \dfrac{(10) + 5}{(10) + 10}$

$= \dfrac{15}{20}$

$\lim\limits_{x \to 10} \dfrac{x^2 - 5x - 50}{x^2 - 100} = \dfrac{3}{4}$

#5) Use the graph of the piecewise function to state each of the limits. Then decide if the function is continuous at x = -300. If not, state why.



a. $\lim\limits_{x \to -300^-} f(x) = 6$

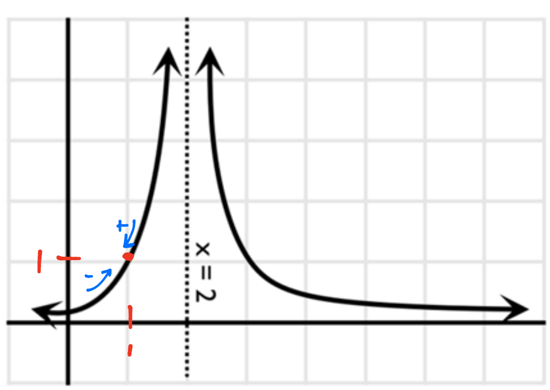
b. $\lim\limits_{x \to -300^+} f(x) = 6$

c. $\lim\limits_{x \to -300} f(x) = 6$

d. Is the function continuous at x = -300? If not, why not?

No, $\lim\limits_{x \to -300} f(x) \neq f(-300)$

#6) Use the graph to state the limit.



$\lim\limits_{x \to 1} f(x) = 1$

#7) Find $f'(x)$ by using the definition of the derivative.

$$f(x) = \frac{x}{x^2+9}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h)}{(x+h)^2+9} - \frac{x}{x^2+9}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h)(x^2+9)}{[(x+h)^2+9](x^2+9)} - \frac{x[(x+h)^2+9]}{[(x+h)^2+9](x^2+9)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h)(x^2+9) - x[x^2+2hx+h^2+9]}{[(x+h)^2+9](x^2+9)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cancel{x^3} + \cancel{9x} + hx^2 + 9h - \cancel{x^3} - 2hx^2 - h^2x - \cancel{9x}}{[(x+h)^2+9](x^2+9)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-hx^2 + 9h - h^2x}{[(x+h)^2+9](x^2+9)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{h(-x^2+9-hx)}{[(x+h)^2+9](x^2+9)} \cdot \frac{1}{h}}{h \cdot \frac{1}{h}}$$

$$= \lim_{h \to 0} \frac{-x^2+9-hx}{[(x+h)^2+9](x^2+9)}$$

$$= \frac{-x^2+9-(0)x}{[(x+(0))^2+9](x^2+9)}$$

$$= \frac{-x^2+9}{[x^2+9](x^2+9)}$$

$$f'(x) = \frac{-x^2+9}{(x^2+9)^2}$$

#8) Find the equation for the tangent line to the curve $f(x) = -4x^2 - 62x + 8$ at $x = -3$. Write the answer in slope-intercept form.

Point @ $x=-3$

$f(-3) = -4(-3)^2 - 62(-3) + 8$

$f(-3) = -4(9) + 186 + 8$

$f(-3) = -36 + 194$

$f(-3) = 158$

$(-3, 158)$

m @ $x=-3$

$f'(x) = -8x - 62$

$f'(-3) = -8(-3) - 62$

$f'(-3) = 24 - 62$

$f'(-3) = -38$

$m = -38$

Point-slope form

$y - y_1 = m(x - x_1)$

$y - (158) = -38(x - (-3))$

$y - 158 = -38x - 114$

$y = -38x + 44$

#9) If $h(x) = (3x^3 - 7)^5(9x^2 + 8)^4$ find $\frac{dh}{dx}$.

$h'(x) = \left[(3x^3-7)^5\right]'(9x^2+8)^4 + (3x^3-7)^5\left[(9x^2+8)^4\right]'$

$= 5(3x^3-7)^4(3x^3-7)'(9x^2+8)^4 + (3x^3-7)^5\, 4(9x^2+8)^3(9x^2+8)'$

$= 5(3x^3-7)^4(9x^2)(9x^2+8)^4 + (3x^3-7)^5\, 4(9x^2+8)^3(18x)$

$= 9x(3x^3-7)^4(9x^2+8)^3\left[5(x)(9x^2+8) + 4(3x^3-7)2\right]$

$= 9x(3x^3-7)^4(9x^2+8)^3\left[45x^3 + 40x + 24x^3 - 56\right]$

$= 9x(3x^3-7)^4(9x^2+8)^3(69x^3 + 40x - 56)$

#10) If $f(x) = \frac{(6x^2+8)^5}{(x^2+1)^3}$ find $\left.\frac{df}{dx}\right|_{x=1}$

$$\frac{df}{dx} = \frac{[(6x^2+8)^5]'(x^2+1)^3 - (6x^2+8)^5[(x^2+1)^3]'}{[(x^2+1)^3]^2}$$

$$= \frac{5(6x^2+8)^4(6x^2+8)'(x^2+1)^3 - (6x^2+8)^5 3(x^2+1)^2(x^2+1)'}{(x^2+1)^6}$$

$$= \frac{5(6x^2+8)^4(12x)(x^2+1)^3 - (6x^2+8)^5 3(x^2+1)^2(2x)}{(x^2+1)^6}$$

$$= \frac{6x(6x^2+8)^4(x^2+1)^2[10(x^2+1) - (6x^2+8)]}{(x^2+1)^6}$$

$$= \frac{6x(6x^2+8)^4(x^2+1)^2[10x^2+10 - 6x^2 - 8]}{(x^2+1)^6}$$

$$\frac{df}{dx} = \frac{6x(6x^2+8)^4\cancel{(x^2+1)^2}[4x^2+2]}{(x^2+1)^{\cancel{6}\,4}}$$

$$\frac{df}{dx} = \frac{6x(6x^2+8)^4[4x^2+2]}{(x^2+1)^4}$$

$$\left.\frac{df}{dx}\right|_{x=1} = \frac{6(1)(6(1)^2+8)^4[4(1)^2+2]}{((1)^2+1)^4}$$

$$= \frac{6(6(1)+8)^4[4(1)+2]}{(1+1)^4}$$

$$= \frac{6(14)^4(4+2)}{(2)^4}$$

$$= \frac{6(14)^4(6)}{2^4}$$

$$\left.\frac{df}{dx}\right|_{x=2} = 86{,}436$$

#11) If $h(x) = 6x^2 - x + 7$ find $\frac{d^2h}{dx^2}$

$$\frac{dh}{dx} = 12x - 1$$

$$\frac{d^2h}{dx^2} = 12$$

#12) If $f(x) = \frac{1}{x}$ find $f'''(x)$

$f(x) = x^{-1}$

$f'(x) = -x^{-2}$

$f''(x) = 2x^{-3}$

$f'''(x) = -6x^{-4}$

$f'''(x) = \frac{-6}{x^4}$

#13) If $h(x) = (9x^2 - 3x + 1)^8$ find $h'(x)$

$h'(x) = 8(9x^2-3x+1)^7 (9x^2-3x+1)'$

$h'(x) = 8(9x^2-3x+1)^7(18x-3)$

#14) If $g(x) = \frac{1}{\sqrt[8]{(5x^3-8)^7}}$ find $g'(x)$

$g(x) = (5x^3-8)^{-7/8}$

$g'(x) = -\frac{7}{8}(5x^3-8)^{-15/8}(5x^3-8)'$

$= -\frac{7}{8}(5x^3-8)^{-15/8}(15x^2)$

$g'(x) = \frac{-105x^2}{8\sqrt[8]{(5x^3-8)^{15}}}$

#15) If $k(x) = (3x - 11)^8(2x+1)^5$ find $k'(x)$

$$k'(x) = \left[(3x-11)^8\right]'(2x+1)^5 + (3x-11)^8\left[(2x+1)^5\right]'$$

$$= 8(3x-11)^7(3x-11)'(2x+1)^5 + (3x-11)^8\,5(2x+1)^4(2x+1)'$$

$$= 8(3x-11)^7(3)(2x+1)^5 + (3x-11)^8\,5(2x+1)^4(2)$$

$$= 2(3x-11)^7(2x+1)^4\left[4(3)(2x+1) + 5(3x-11)\right]$$

$$= 2(3x-11)^7(2x+1)^4\left[24x+12+15x-55\right]$$

$$k'(x) = 2(3x-11)^7(2x+1)^4\left[39x-43\right]$$

#16) If $w(x) = \left(\frac{x^2-8}{x^3+1}\right)^3$ find $w'(x)$

$$w'(x) = 3\left(\frac{x^2-8}{x^3+1}\right)^2 \cdot \left(\frac{x^2-8}{x^3+1}\right)'$$

$$= 3\,\frac{(x^2-8)^2}{(x^3+1)^2} \cdot \frac{(x^2-8)'(x^3+1) - (x^2-8)(x^3+1)'}{(x^3+1)^2}$$

$$= 3\,\frac{(x^2-8)^2}{(x^3+1)^2} \cdot \frac{(2x)(x^3+1) - (x^2-8)(3x^2)}{(x^3+1)^2}$$

$$= 3\,\frac{(x^2-8)^2}{(x^3+1)^2} \cdot \frac{2x^4+2x-3x^4+24x^2}{(x^3+1)^2}$$

$$= 3\,\frac{(x^2-8)^2}{(x^3+1)^2} \cdot \frac{-x^4+24x^2+2x}{(x^3+1)^2}$$

$$w'(x) = \frac{-3x(x^2-8)^2(x^3-24x-2)}{(x^3+1)^4}$$

#17) While in the hospital recovering from food poisoning reaction, George finds himself with nothing but time on his hands. That's right; he decides to steal all the clocks in the hospital. George's revenue from selling *x* hospital clocks is $R(x) = \sqrt{x^3 + 3x}$ dollars (for $0 \le x \le 10$).

$\boxed{R(x) = \$ \quad x = \text{clocks}}$

a. Find the marginal average revenue function.

$R(x) = (x^3+3x)^{\frac{1}{2}}$

$AR(x) = \frac{R(x)}{x}$

$AR(x) = \frac{(x^3+3x)^{\frac{1}{2}}}{x}$

$MAR(x) = \frac{[(x^3+3x)^{\frac{1}{2}}]'(x) - (x^3+3x)^{\frac{1}{2}}(x)'}{x^2}$

$= \frac{\frac{1}{2}(x^3+3x)^{-\frac{1}{2}}(x^3+3x)'x - (x^3+3x)^{\frac{1}{2}}(1)}{x^2}$

$= \frac{\frac{1}{2}(x^3+3x)^{-\frac{1}{2}}(3x^2+3)x - (x^3+3x)^{\frac{1}{2}}}{x^2}$

$= \frac{(x^3+3x)^{-\frac{1}{2}}\left[\frac{1}{2}(3x^3+3x) - (x^3+3x)\right]}{x^2}$

$= \frac{\frac{1}{\sqrt{x^3+3x}}\left[\frac{3}{2}x^3 + \frac{3}{2}x - x^3 - 3x\right]}{x^2}$

$= \frac{\frac{1}{2}x^3 - \frac{3}{2}x}{\sqrt{x^3+3x}} \cdot \frac{1}{x^2} = \frac{\frac{1}{2}\cancel{x}(x^2-3)}{\sqrt{x^3+3x}} \cdot \frac{1}{x^{\cancel{2}}}$

$\frac{x^2-3}{2x\sqrt{x^3+3x}}$

MAR(x) =

b. Evaluate the marginal average revenue function at 5 and interpret this answer.

$MAR(5) = \frac{(5)^2 - 3}{2(5)\sqrt{(5)^3+3(5)}}$

$= \frac{25-3}{10\sqrt{125+15}}$

$= \frac{22}{10\sqrt{140}}$

$= \frac{11}{5\sqrt{140}}$

$MAR(5) \approx .19$

MAR(5) = \$.19/clock

Interpretation =

After George has sold 5 clocks, his average revenue per clock is increasing by 19 cents per clock sold.

#18) With his newfound fortune of \$32.09, George has a rare wise thought. He deposits the money in an account earning interest at *r* percent annually, after 3 years its value will be $V(r) = 32.09(1 + 0.01r)^3$ dollars.

Find $V'(8)$ and interpret this answer.

$\boxed{V = \text{value } \$ \quad r = \% \text{ interest}}$

$V'(r) = 32.09(3)(1+0.01r)^2(1+0.01r)'$

$= 96.27(1+0.01r)^2(0.01)$

$V'(r) = .9627(1+0.01r)^2$

$V'(8) = .9627(1+0.01(8))^2$

$= .9627(1+0.08)^2$

$= .9627(1.08)^2$

$V'(8) \approx \$1.12/\%$

$V'(8)$ = \$1.12/%

Interpretation =

When the account earns 8% interest annually, the value of the money is increasing by \$1.12 per percent increase.

#19) All good plans seem to fail when George is involved. Instead of investing his \$32.09, George went to Burger King and blew his money on Whoppers. Sadly for Burger King, George did a little more than give them money; he also gave them fleas. The number of fleas at this Burger King x days from George infecting them is predicted to be $P(x) = \sqrt[4]{x^2+1}$ million fleas for $1 \le x \le 5$.

$P(x)$ = millon fleas
x = days

a. Find $P'(3)$ and interpret this answer.

$P(x) = (x^2+1)^{1/4}$

$$\boxed{\begin{aligned} P'(x) &= \tfrac{1}{4}(x^2+1)^{-3/4}(x^2+1)' \\ &= \tfrac{1}{4}(x^2+1)^{-3/4}(2x) \\ P'(x) &= \frac{x}{2\sqrt[4]{(x^2+1)^3}} \end{aligned}}$$

$$P'(3) = \frac{3}{2\sqrt[4]{((3)^2+1)^3}} = \frac{3}{2\sqrt[4]{(9+1)^3}} = \frac{3}{2\sqrt[4]{10^3}} = \frac{3}{2\sqrt[4]{1000}}$$

$P'(3) =$ .266742 million fleas/day

Interpretation =

Three days after George gives BK fleas, the flea population is growing by 266,742 fleas per day.

b. Assume $P''(3) = -.031120$. Interpret this answer.

Interpretation =

Three days after George gives BK fleas, the flea population growth rate is decreasing by 31,120 fleas per day each day.

#20) After a delicious lunch, George continues on his journey to California. Brainless and penniless, he hops on a super train. At t hours on the train George is $s(t) = 12t^2 - t^3$ miles due north of its starting point (for $0 \le t \le 12$).

a. Find George's velocity at time $t = 3$ hours. Write your answer as a sentence.

$v(t) = 24t - 3t^2$

$$\boxed{\begin{aligned} v(3) &= 24(3) - 3(3)^2 \\ &= 72 - 3(9) \\ &= 72 - 27 \\ v(3) &= 45 \text{ mph North} \end{aligned}}$$

At 3 hours of riding on the train, George's velocity is 45 miles per hour north.

b. Find his velocity at time $t = 8$ hours. Write ~~your answer as a sentence.~~

$$\boxed{\begin{aligned} v(8) &= 24(8) - 3(8)^2 \\ &= 192 - 3(64) \\ &= 192 - 192 \\ v(8) &= 0 \text{ mph} \end{aligned}}$$

At 8 hours of riding on the train, George's velocity is 0 miles per hour. (we can't assign a direction if magnitude is 0)

c. Find his acceleration at time $t = 8$ hours. Write your answer as a sentence.

$a(t) = 24 - 6t$

$$\boxed{\begin{aligned} a(8) &= 24 - 6(8) \\ &= 24 - 48 \\ a(8) &= -24 \text{ mph}^2 \end{aligned}}$$

At 8 hours of riding on the train, George's velocity is decreasing by 24 miles per hour each hour.

#21) Six hours after the train headed north, the crewmen found George. And what does a crew do with a stow away? They roll out the cannon. George's height can be found by the function

$$h(t) = -0.025(t - 15.5)^2 + 6.00625$$

where *t* is the number of seconds after George has been shot out of the cannon and *h(t)* is the height in feet.

a. How long will it take for George to hit the ground? Use a sentence answer.

$$0 = -0.025(t-15.5)^2 + 6.00625$$
$$-6.00625 = -0.025(t-15.5)^2$$
$$240.25 = (t-15.5)^2$$
$$\pm 15.5 = t - 15.5$$
$$15.5 \pm 15.5 = t$$
$$0,\ 31 = t$$

Once George is shot out of the cannon, he will hit the ground in 31 seconds.

b. With what velocity will George hit the ground? Use a sentence answer.

$$h(t) = -0.025(t-15.5)^2 + 6.00625$$
$$v(t) = -0.050(t-15.5)\cdot(t-15.5)'$$
$$v(t) = -0.05(t-15.5)\cdot(1)$$
$$v(t) = -0.05t + 0.775$$
$$v(31) = -0.05(31) + 0.775$$
$$= -1.55 + 0.775$$
$$= -.775 \text{ ft/sec up}$$
$$v(31) = .775 \text{ ft/sec down}$$

Thirty one seconds after being shot out of a cannon, George's impact velocity is 0.755 feet per second down.

#22) Dusting himself off, resilient George finds \$50 laying in the dirt. He also finds himself no more closer to California than when he started his journey. In fact, he now finds himself in Canada. He gets another brilliant idea: ride a rocket to California. Because Canadian law is so lax, anyone can buy a rocket at any time. No questions asked. Oh, and every rocket in Canada costs exactly \$50. George walks into a CVS and buys a rocket, straps himself to it and lights it. The rocket rises $s(t) = 8t^{5/2}$ feet in t seconds.

a. Find George's velocity after 25 seconds. Use a sentence answer.

$$v(t) = 20t^{3/2}$$
$$v(25) = 20\sqrt{(25)^3}$$
$$= 20(5)^3$$
$$= 20(125)$$
$$v(25) \approx 2500 \text{ ft/sec up}$$

Twenty five seconds after blast off, George's velocity is 2500 feet per second traveling up.

b. Find George's acceleration after 25 seconds. Use a sentence answer.

$$a(t) = 30t^{\frac{1}{2}}$$
$$a(25) = 30\sqrt{25}$$
$$= 30(5)$$
$$a(25) = 150 \text{ ft/sec}$$

Twenty five seconds after blast off, George's velocity is increasing by 150 feet per second each second.

#23) $\frac{d}{dx}[\sin(x^2+5)]$ $= \cos(x^2+5) \cdot \frac{d}{dx}(x^2+5)$

$= \cos(x^2+5) \cdot (2x)$

$\frac{d}{dx}\left[\sin(x^2+5)\right] = 2x \cos(x^2+5)$

#24) $\frac{d}{dx}\left[\frac{\tan(x)}{\cot(x)}\right]$ $= \frac{d}{dx}\left[\tan(x) \cdot \tan(x)\right]$

$= \frac{d}{dx}\left[\tan(x)\right]^2$

$= 2\left[\tan(x)\right] \cdot \left[\tan(x)\right]'$

$\frac{d}{dx}\left[\frac{\tan(x)}{\cot(x)}\right] = 2\tan(x) \cdot \sec^2(x)$

#25) $\frac{d}{dx}[(7x^2-4)\cos(x)]$

$= \frac{d}{dx}(7x^2-4)\cos(x) + (7x^2-4)\frac{d}{dx}\cos(x)$

$= (14x)\cos(x) + (7x^2-4)(-\sin(x))$

$\frac{d}{dx}\left[(7x^2-4)\cos(x)\right] = 14x\cos(x) - (7x^2-4)\sin(x)$

#26) $\frac{d}{dx}[\sin^2(x^2+5)+\cos^2(x^2+5)]$ $= \frac{d}{dx}\left[1\right]$

$\frac{d}{dx}\left[\sin^2(x^2+5)+\cos^2(x^2+5)\right] = 0$