Find the limits by using any method. Then, circle which method you used.

#1) $\lim\limits_{x\to 9} \dfrac{\frac{1}{3x}-4}{2x-18}$ = dne

Graphing

Table

Substitution

$x \to 9^-$ $\quad$ $9^+ \leftarrow x$

x	8.9	8.99	8.999	9.001	9.01	9.1
y	19.8	198.2	1981.5	-1981	-198.2	-19.82

$y \to \infty$ $\quad$ $-\infty \leftarrow y$

#2) $\lim\limits_{x\to -2} \dfrac{x^2+4x+4}{x^2+2x} = \lim\limits_{x\to -2} \dfrac{(x+2)^2}{x(x+2)}$

Graphing

Table

Substitution

$= \lim\limits_{x\to -2} \dfrac{x+2}{x}$

$= \dfrac{(-2)+2}{(-2)}$

$= \dfrac{0}{-2}$

$\lim\limits_{x\to -2} \dfrac{x^2+4x+4}{x^2+2x} = 0$

#3) $\lim\limits_{x\to 2}(x^2-4x+9) = (2)^2-4(2)+9$

Graphing

Table

Substitution

$= 4-8+9$

$= 4+1$

$\lim\limits_{x\to 2}(x^2-4x+9) = 5$

FACTOR

$2x^2-15x+18$

$= 2x^2 - 3x + -12x + 18$

$= (2x^2-3x) + (-12x+18)$

$= x(2x-3) + -6(2x-3)$

$= (2x-3)(x-6)$

#4) $\lim\limits_{x\to 6} \dfrac{2x^2-15x+18}{x^2-6x} = \lim\limits_{x\to 6} \dfrac{(2x-3)(x-6)}{x(x-6)}$

Graphing

Table

Substitution

$= \lim\limits_{x\to 6} \dfrac{2x-3}{x}$

$= \dfrac{2(6)-3}{(6)}$

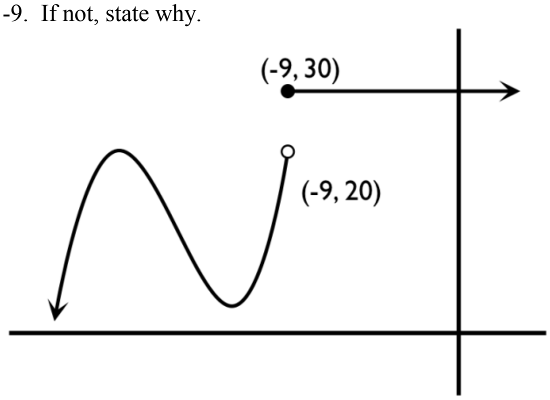
$= \dfrac{12-3}{6}$

$= \dfrac{9}{6}$

$\lim\limits_{x\to 6} \dfrac{2x^2-15x+18}{x^2-6x} = \dfrac{3}{2}$

mult	Add
$2x^2(18)$	$-15x$
$-x(-36x)$	$-37x$
$-2x(-18x)$	$-20x$
$-3x(-12x)$	$-15x$

#5) Use the graph of the piecewise function to state each of the limits. Then decide if the function is continuous at x = -9. If not, state why.



a. $\lim\limits_{x\to -9^-} f(x) =$ 20

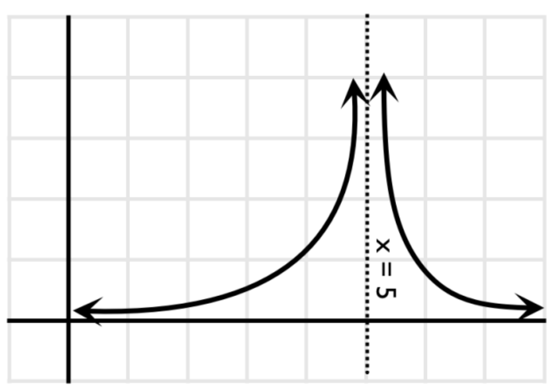
b. $\lim\limits_{x\to -9^+} f(x) =$ 30

c. $\lim\limits_{x\to -9} f(x) =$ dne

d. Is the function continuous at x = -9? If not, why not?

No $\lim\limits_{x\to -9} f(x) =$ dne

#6) Use the graph to state the limit.



$\lim\limits_{x\to 5} f(x) = \infty$, dne.

#7) Find $f'(x)$ by using the definition of the derivative.

$$f(x) = \frac{2}{x-2}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2}{(x+h)-2} - \frac{2}{x-2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2(x-2)}{(x+h-2)(x-2)} - \frac{2(x+h-2)}{(x-2)(x+h-2)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(\cancel{2x}-\cancel{4}) - (\cancel{2x}+2h-\cancel{4})}{(x+h-2)(x-2)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-2h}{(x+h-2)(x-2)}}{h} \cdot \frac{\frac{1}{h}}{\frac{1}{h}}$$

$$= \lim_{h\to 0} \frac{-2}{(x+h-2)(x-2)}$$

$$= \frac{-2}{[x+(0)-2][x-2]}$$

$$f'(x) = \frac{-2}{[x-2]^2}$$

#8) Find the equation for the tangent line to the curve $f(x) = 2x^2 - x + 11$ at $x = -3$. Write the answer in slope-intercept form.

Point @ $x = -3$

$f(-3) = 2(-3)^2 - (-3) + 11$
$= 2(9) + 3 + 11$
$= 18 + 14$
$f(-3) = 32$
$(-3, 32)$

m @ $x = -3$

$f'(x) = 4x - 1$
$f'(-3) = 4(-3) - 1$
$= -12 - 1$
$f'(-3) = -13$
$m = -13$

Point - Slope Form

$y - y_1 = m(x - x_1)$
$y - (32) = -13(x - (-3))$
$y - 32 = -13x - 39$
$y = -13x - 7$

#9) If $h(x) = (x^2 + 2x)(2x + 1)$ find $\frac{dh}{dx}$.

$\frac{dh}{dx} = \frac{d}{dx}(x^2 + 2x) \cdot (2x + 1) + (x^2 + 2x) \frac{d}{dx}(2x + 1)$
$= (2x + 2)(2x + 1) + (x^2 + 2x)(2)$
$= 4x^2 + 6x + 2 + 2x^2 + 4x$
$\frac{dh}{dx} = 6x^2 + 10x + 2$

#10) If $f(x) = \frac{x^4+x^2+1}{x^2+1}$ find $\left.\frac{df}{dx}\right|_{x=-2}$

$$\frac{df}{dx} = \frac{\frac{d}{dx}(x^4+x^2+1)\cdot(x^2+1) - (x^4+x^2+1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(4x^3+2x)(x^2+1) - (x^4+x^2+1)(2x)}{(x^2+1)^2}$$

$$= \frac{4x^5+4x^3+2x^3+2x-2x^5-2x^3-2x}{(x^2+1)^2}$$

$$= \frac{2x^5+4x^3}{(x^2+1)^2}$$

$$\frac{df}{dx} = \frac{2x^3(x^2+2)}{(x^2+1)^2}$$

$$\left.\frac{df}{dx}\right|_{x=-2} = \frac{2(-2)^3[(-2)^2+2]}{[(-2)^2+1]^2}$$

$$= \frac{2(-8)[4+2]}{[4+1]^2}$$

$$= \frac{-16[6]}{[5]^2}$$

$$\left.\frac{df}{dx}\right|_{x=-2} = \frac{-96}{25}$$

#11) If $h(x) = 6x^7 - 8x^4 + 4$ find $\frac{d^2h}{dx^2}$

$$\frac{dh}{dx} = 42x^6 - 32x^3$$

$$\frac{d^2h}{dx^2} = 252x^5 - 96x^2$$

#12) If $f(x) = \frac{5}{x^6}$ find $f''(x)$

$f(x) = 5x^{-6}$

$f'(x) = -30x^{-7}$

$f''(x) = 210x^{-8}$

$f''(x) = \frac{210}{x^8}$

#13) If $h(x) = (3x^2 + 5x + 2)^4$ find $h'(x)$

$h'(x) = 4(3x^2+5x+2)^3 (3x^2+5x+2)'$

$h'(x) = 4(3x^2+5x+2)^3 (6x+5)$

#14) If $g(x) = \frac{1}{\sqrt[3]{(3x-1)^2}}$ find $g'(x)$

$g(x) = (3x-1)^{-2/3}$

$g'(x) = -\frac{2}{3}(3x-1)^{-5/3} (3x-1)'$

$g'(x) = \frac{-2}{3\sqrt[3]{(3x-1)^5}} (3)$

$g'(x) = \frac{-2}{\sqrt[3]{(3x-1)^5}}$

#15) If $k(x) = 3x^2(2x+1)^5$ find $k'(x)$

$$k'(x) = (3x^2)'(2x+1)^5 + 3x^2\left[(2x+1)^5\right]'$$

$$= 6x(2x+1)^5 + 3x^2(5)(2x+1)^4(2x+1)'$$

$$= 6x(2x+1)^5 + 3x^2(5)(2x+1)^4(2)$$

$$= 6x(2x+1)^4\left[(2x+1) + x(5)\right]$$

$$k'(x) = 6x(2x+1)^4(7x+1)$$

#16) If $w(x) = \left(\frac{x-1}{x+1}\right)^5$ find $w'(x)$

$$w'(x) = 5\left(\frac{x-1}{x+1}\right)^4\left(\frac{x-1}{x+1}\right)'$$

$$= 5\frac{(x-1)^4}{(x+1)^4}\cdot\frac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2}$$

$$= \frac{5(x-1)^4}{(x+1)^4}\cdot\frac{(1)(x+1)-(x-1)(1)}{(x+1)^2}$$

$$= \frac{5(x-1)^4}{(x+1)^4}\cdot\frac{x+1-x+1}{(x+1)^2}$$

$$= \frac{5(x-1)^4}{(x+1)^6}\cdot 2$$

$$w'(x) = \frac{10(x-1)^4}{(x+1)^6}$$

#17) While in flight on his rocket, George has another brilliant idea: he wants to sell tweezers to lumberjacks. Shortly after, he blacks out. During his comatose state, he has a vision consisting of a midget Happy Gilmore riding a tricycle on the beach. Happy looks at George and says, "Your cost function is $C(x) = \sqrt{4x^2 + 900}$ dollars, where *x* is the number of tweezers produced."

$\boxed{\begin{array}{l} C = \$ \\ x = \text{tweezers} \end{array}}$

a. Find the marginal cost function:

$$MC(x) = \frac{1}{2}(4x^2+900)^{-\frac{1}{2}}(4x^2+900)'$$

$$= \frac{8x}{2\sqrt{4x^2+900}}$$

$$MC(x) = \frac{4x}{\sqrt{4x^2+900}}$$

$MC(x) = \dfrac{4x}{\sqrt{4x^2+900}}$

b. Evaluate the marginal cost function at 30 and interpret this answer.

$$MC(30) = \frac{4(30)}{\sqrt{4(30)^2+900}}$$

$$= \frac{120}{\sqrt{4(900)+900}}$$

$$= \frac{120}{\sqrt{3600+900}}$$

$$= \frac{120}{\sqrt{4500}}$$

$$MC(30) \approx \$1.79/\text{tweezers}$$

$MC(30) = \$1.79/\text{tweezers}$

Interpretation =

When 30 tweezers have been made, the cost to make the next tweezer is \$1.79.

#18) An unconscious George laments his social status. A person's social status (rated on a scale where 100 indicates the status of a college graduate) depends on years of education. A person with *e* years of education has a social status of $S(e) = 0.22(e+4)^{2.1}$. George's highest level of education is 4th grade.

Find the instantaneous rate of change of George's social status at 2nd grade and interpret this answer.

$S(e) =$ social status points

$e =$ year of ed

$$\boxed{\begin{aligned} S'(e) &= 0.22(2.1)(e+4)^{1.1}(e+4)' \\ &= .462(e+4)^{1.1}(1) \\ S'(e) &= .462(e+4)^{1.1} \end{aligned}}$$

$$\boxed{\begin{aligned} S'(2) &= .462((2)+4)^{1.1} \\ &= .462(6)^{1.1} \\ S'(2) &\approx 3.32 \frac{\text{social status points}}{\text{year of ed}} \end{aligned}}$$

$S'(2) = 3.32 \dfrac{\text{social status points}}{\text{year of ed}}$

Interpretation =

When George had 2 years of education, his social status is increasing by 3.32 social status points per year of education.

#19) As the rocket continues to rise George gets colder and colder. On this particular day, the temperature of the atmosphere t miles from the earth's crust is $T(t) = 65 - \frac{t^2}{4}$ (for $t \geq 16$).

$T = °F$
$t = \text{miles}$

$T(t) = 65 - \frac{t^2}{4}$

a. Find $T'(10)$ and interpret this answer.

$T'(t) = -\frac{1}{2}t$

$T'(10) = -\frac{1}{2}(10)$

$T'(10) = -5°F/\text{mile}$

$T'(10) =$ $-5°F/\text{mile}$

Interpretation =

When George is 10 miles from the earth's crust, the temperature is decreasing by 5°F per mile.

b. Find $T''(10)$ and interpret this answer.

$T''(t) = -\frac{1}{2}$

$T''(10) =$ $-\frac{1}{2}\ °F/\text{mile}^2$

Interpretation =

When George is 10 miles from the earth's crust, the rate at which the temperature is changing is decreasing by 1/2°F per mile each mile.

#20) When the rocket finally crashes and all the dust settles, it does so in Fairbanks, Alaska. Not wanting to make the same mistake he made earlier, George stows away on a train traveling *west*. After t hours George is $s(t) = 24t^2 - 2t^3$ miles due west of his starting point (for $0 \leq t \leq 12$).

$s = \text{miles}$
$t = \text{hours}$

a. Find the train's velocity at time $t = 4$ hours. Write your answer as a sentence.

$v(t) = 48t - 6t^2$

$$\boxed{\begin{aligned} v(4) &= 48(4) - 6(4)^2 \\ &= 192 - 6(16) \\ &= 192 - 96 \\ v(4) &= 96 \text{ mph West} \end{aligned}}$$

Four hours after riding in the train, George's velocity is 96 mph heading west.

b. Find the train's velocity at time $t = 10$ hours. Write your answer as a sentence.

$$\boxed{\begin{aligned} v(10) &= 48(10) - 6(10)^2 \\ &= 480 - 6(100) \\ &= 480 - 600 \\ v(4) &= {}^{-}120 \text{ mph West} \\ v(10) &= 120 \text{ mph East} \end{aligned}}$$

Ten hours after riding in the train, George's velocity is 120 mph heading east.

c. Find the train's acceleration at time $t = 1$ hours. Write your answer as a sentence.

$a(t) = 48 - 12t$

$a(1) = 48 - 12(1)$

$a(1) = 48 - 12$

$a(1) = 36 \text{ mph}^2$

One hour after riding in the train, George's velocity is increasing by 36 mph each hour.

#21) While on the train George finds a marble. Not just any marble. This is the same marble that caused George's parents to divorce. After a long fight, filled with hurtful words and explosive blows, George finally gets the upper hand in the heated battle. With all his might, he is able to push the marble out of the train while on Cushman Street Bridge. The height above the water *t* seconds after the marble is discarded is $s(t) = 128 - 16t^2$ feet (neglecting air resistance).

s = feet above H_2O
t = seconds

a. How long will it take to hit the water? Use a sentence answer.

$0 = 128 - 16t^2$

$16t^2 = 128$

$t^2 = 8$

$t = \pm 2.8$ (−2.8 makes no sense)

$t = 2.8$ seconds

Once thrown from the moving train, it will take the relationship wrecking marble 2.8 seconds to hit the water.

b. With what velocity will the marble hit the water? Use a sentence answer.

$v(t) = -32t$

$v(2.8) = -32(2.8)$

$v(2.8) = -89.6$ ft/sec

When the marble hits the water 2.8 seconds after being thrown from a train, it's impact velocity is 89.6 feet per second and is headed down.

#22) When the train stops, George runs over to the sandy beach with unbridled excitement. He finally made it to California, or so he *thought*. When he hears a woman yell, "I can see Russia from my house," he realizes he is in fact not in California, but Alaska. He swiftly commandeers a Hummer and heads southeast. After *t* hours George is a distance $s(t) = 60t + \frac{100}{t+3}$ miles from his starting point.

s(t) = miles
t = hours

a. Find the velocity after 2 hours. Use a sentence answer.

$s(t) = 60t + 100(t+3)^{-1}$

$$\boxed{\begin{aligned} v(t) &= 60 - 100(t+3)^{-2}(t+3)' \\ v(t) &= 60 - \frac{100}{(t+3)^2}(1) \\ v(t) &= 60 - \frac{100}{(t+3)^2} \end{aligned}}$$

$v(2) = 60 - \frac{100}{[(2)+3]^2}$

$= 60 - \frac{100}{[5]^2}$

$= 60 - \frac{100}{25}$

$= 60 - 4$

$v(2) = 56$ m/h

At 2 hours, George's velocity is 56 miles per hour heading southeast.

b. Find the acceleration after 2 hours. Use a sentence answer.

$v(t) = 60 - 100(t+3)^{-2}$

$$\boxed{\begin{aligned} a(t) &= 200(t+3)^{-3} \\ a(t) &= \frac{200}{(t+3)^3} \end{aligned}}$$

$a(3) = \frac{200}{[(3)+3]^3}$

$= \frac{200}{(9)^3}$

$= \frac{200}{729}$

$a(3) = .27$ mph/h

At 2 hours, George's velocity is increasing by 0.27 miles per hour each hour.

#23) $\frac{d}{dx}[\csc(x)\cos(x)] = \frac{d}{dx}\left[\frac{1}{\sin(x)}\cos(x)\right]$

$= \frac{d}{dx}\left[\cot(x)\right]$

$\frac{d}{dx}\left[\csc(x)\cos(x)\right] = -\csc^2(x)$

#24) $\frac{d}{dx}\left[\frac{\tan^2(x)+\sin^2(x)+\cos^2(x)}{\frac{1}{\cos^2(x)}}\right]$

$= \frac{d}{dx}\left[\frac{\tan^2(x)+1}{\sec^2(x)}\right]$

$= \frac{d}{dx}\left[\frac{\sec^2(x)}{\sec^2(x)}\right]$

$= \frac{d}{dx}\left[1\right]$

$\frac{d}{dx}\left[\frac{\tan^2(x)+\sin^2(x)+\cos^2(x)}{\frac{1}{\cos^2(x)}}\right] = 0$

#25) $\frac{d}{dx}[\sin^3(5x) + \cos^2(5x)]$

$= 3\sin^2(5x)\cdot\frac{d}{dx}(5x) + 2\cos(5x)\cdot\frac{d}{dx}(5x)$

$= 3\sin^2(5x)\cdot(5) + 2\cos(5x)\cdot(5)$

$= 15\sin^2(5x) + 10\cos(5x)$

#26) $\frac{d}{dx}[\cos(\sin(x^2))]$

$= -\sin\left[\sin(x^2)\right]\cdot\frac{d}{dx}\sin(x^2)$

$= -\sin\left[\sin(x^2)\right]\cos(x^2)\cdot\frac{d}{dx}(x^2)$

$= -\sin\left[\sin(x^2)\right]\cos(x^2)\,(2x)$

$= -2x\sin\left[\sin(x^2)\right]\cos(x^2)$