Basic Integration
Review Chapter 8

Use the Fundamental Theorem of Calculus to find the area under the curve.

#1) $f(x) = \frac{\tan(x)\cos(x)}{\sin(x)}$ from x = 4 to x = 10

$$A = \int_4^{10} \frac{\frac{\sin(x)}{\cancel{\cos(x)}} \cdot \cancel{\cos(x)}}{\sin(x)} dx = \int_4^{10} \frac{\cancel{\sin(x)}}{\cancel{\sin(x)}} dx$$

$$= \int_4^{10} 1\,dx = x \Big|_4^{10} = (10) - (4)$$

$$A = 6 \text{ un}^2$$

#2) $f(x) = x$ from x = 0 to x = 4

$$A = \int_0^4 x\,dx$$

$$= \frac{1}{2}x^2 \Big|_0^4$$

$$= \frac{1}{2}(4)^2 - \frac{1}{2}(0)^2$$

$$= \frac{1}{2}(16) - \frac{1}{2}(0)$$

$$= 8 - 0$$

$$A = 8 \text{ un}^2$$

#3) $f(x) = e^x$ from x = 0 to x = 1

$$A = \int_0^1 e^x\,dx$$

$$= e^x \Big|_0^1$$

$$= e^{(1)} - e^{(0)}$$

$$A = (e - 1) \text{ un}^2$$

#4) $f(x) = 9 - 3\sqrt{x}$ from x = 0 to x = 9.

$$A = \int_0^9 (9 - 3x^{\frac{1}{2}})dx$$

$$= \left(9x - 2x^{\frac{3}{2}}\right)\Big|_0^9$$

$$= \left[9(9) - 2(\sqrt{9})^3\right] - \left[9(0) - 2(\sqrt{0})^3\right]$$

$$= \left[81 - 2(3)^3\right] - \left[0 - 0\right]$$

$$= \left[81 - 2(27)\right] - \left[0\right]$$

$$= 81 - 54$$

$$A = 27 \text{ un}^2$$

#5) $f(x) = \frac{1}{x}$ from x = e to x = e^3.

$$A = \int_e^{e^3} \frac{1}{x}\,dx$$

$$= \ln|x| \Big|_e^{e^3}$$

$$= \ln|e^3| - \ln|e|$$

$$= 3 - 1$$

$$A = 2 \text{ un}^2$$

#6) $\int_0^1 (6x^2 - 4e^{2x})dx$

$$= \left(2x^3 - 2e^{2x}\right)\Big|_0^1$$

$$= \left[2(1)^3 - 2e^{2(1)}\right] - \left[2(0)^3 - 2e^{2(0)}\right]$$

$$= \left[2(1) - 2e^2\right] - \left[0 - 2e^0\right]$$

$$= \left[2 - 2e^2 + 2(1)\right]$$

$$= (4 - 2e^2) \text{un}^2$$

#7) $\int_1^2 \frac{(x+1)^2}{x^2} dx$

$$A = \int_1^2 \frac{x^2 + 2x + 1}{x^2} dx$$

$$= \int_1^2 (1 + 2x^{-1} + x^{-2})dx$$

$$= \left(x + 2\ln|x| - x^{-1}\right)\Big|_1^2$$

$$= \left[(2) + 2\ln|2| - (2)^{-1}\right] - \left[(1) + 2\ln|1| - (1)^{-1}\right]$$

$$= \left[2 + 2\ln|2| - \frac{1}{2}\right] - \left[1 + 2(0) - 1\right]$$

$$= \left[1.5 + 2\ln(2)\right] - \left[0\right]$$

$$= \left[1.5 + 2\ln(2)\right] \text{un}^2$$

$$\approx 2.89 \text{ un}^2$$

#8) An average child of age x years gains weight at the rate of $3.9x^{1/2}$ pounds per year (for the first 16 years). Find the total weight gain from age 1 to age 9.

x = age of child
WG = weight gained

$$WG = \int_1^9 3.9x^{\frac{1}{2}}dx = 3.9\left(\tfrac{2}{3}\right)x^{\frac{3}{2}}\Big|_1^9$$
$$= 2.6(\sqrt{x})^3\Big|_1^9$$
$$= 2.6(\sqrt{(9)})^3 - 2.6(\sqrt{(1)})^3$$
$$= 2.6(3)^3 - 2.6(1)$$
$$= 2.6(27) - 2.6$$
$$= 70.2 - 2.6$$
$$WG = \quad \text{lbs}$$

The total weight gained from 1 to 9 is lbs.

#9) A company's marginal cost function is MC(x) = $8e^{-0.01x}$, where x is the number of units. Find the total cost of the first hundred units. (x = 0 to x = 100).

x = # of units
C = total cost \$

$$C = \int_0^{100} 8e^{-0.01x}dx$$
$$= 8(-100)e^{-0.01x}\Big|_0^{100}$$
$$= (-800e^{-0.01(100)}) - (-800e^{-0.01(0)})$$
$$= -800e^{-1} + 800e^0$$
$$= \frac{-800}{e} + 800$$
$$C \approx \$505.70$$

The first 100 units cost about \$505.70

#10) There are approximately $2.3e^{0.01t}$ million marriages per year in the United States, where t is the number of years since 1995. Assuming that this rate continues, find the average number of marriages from the year 1995 to the year 2005.

t = years since 1995
M = marriages in millions

$$AM = \frac{1}{10-0}\int_0^{10} 2.3e^{0.01t}dt$$
$$= \frac{1}{10}\left[2.3(100)e^{0.01t}\right]\Big|_0^{10}$$
$$= \frac{1}{10}\left[230e^{0.01t}\right]\Big|_0^{10}$$
$$= \frac{1}{10}\left[230e^{0.01(10)} - 230e^{0.01(0)}\right]$$
$$= \frac{1}{10}\left[230e^{.1} - 230e^0\right]$$
$$= \frac{1}{10}\left[230e^{.1} - 230\right]$$
$$M \approx 2.418931 \text{ million marriages}$$

There were about 2,418931 marriages from 1995 to 2005.

#11) What is the difference between an indefinite integral and a definite integral?

Next page

#12) Explain in words what the answer to this problem means. $\int_3^7 x^2 dx$

(note: you do not need to actually solve this problem.)

Next page

#13) Find the area bounded by the curves.

$y = 4x^3$ and $y = 12x^2 - 8x$

Cross? Yes @ 0, 1, 2

$$4x^3 = 12x^2 - 8x$$
$$4x^3 - 12x^2 + 8x = 0$$
$$4x(x^2 - 3x + 2) = 0$$
$$4x(x-2)(x-1) = 0$$
$$4x = 0 \quad x - 2 = 0 \quad x - 1 = 0$$
$$x = 0 \quad x = 2 \quad x = 1$$

Upper/Lower

$\frac{1}{2} \in (0,1)$

$y = 4x^3$	$y = 12x^2 - 8x$
$y = 4(\frac{1}{2})^3$	$y = 12(\frac{1}{2})^2 - 8(\frac{1}{2})$
$y = 4(\frac{1}{8})$	$y = 12(\frac{1}{4}) - 4$
$y = \frac{1}{2}$	$y = 3 - 4$
	$y = -1$
upper	Lower

Upper/Lower

$\frac{3}{2} \in (1,2)$

$y = 4x^3$	$y = 12x^2 - 8x$
$y = 4(\frac{3}{2})^3$	$y = 12(\frac{3}{2})^2 - 8(\frac{3}{2})$
$y = 4(\frac{27}{8})$	$y = 12(\frac{9}{4}) - 4(3)$
$y = \frac{27}{2}$	$y = 3(9) - 12$
	$y = 27 - 12$
	$y = 15$
Lower	upper

$$A = \int_0^1 [(4x^3) - (12x^2 - 8x)]\,dx + \int_1^2 [(12x^2 - 8x) - (4x^3)]\,dx$$
$$= \int_0^1 [4x^3 - 12x^2 + 8x]\,dx + \int_1^2 [-4x^3 + 12x^2 - 8x]\,dx$$
$$= \left[x^4 - 4x^3 + 4x^2\right]\Big|_0^1 + \left[-x^4 + 4x^3 - 4x^2\right]\Big|_1^2$$
$$= [(1)^4 - 4(1)^3 + 4(1)^2] - [(0)^4 - 4(0)^3 + 4(0)^2] + [-(2)^4 + 4(2)^3 - 4(2)^2] - [-(1)^4 + 4(1)^3 - 4(1)^2]$$
$$= [1 - 4 + 4] - [0] + [-16 + 4(8) - 4(4)] - [-1 + 4 - 4]$$
$$= [1] + [-16 + 32 - 16] - [-1]$$
$$= 1 + 0 + 1$$
$$A = 2\text{ un}^2$$

#14) Find the area bounded by the curves.

$y = x^3 + x^2$ and $y = x^2 + x$

Cross? Yes @ -1, 0, 1

$$x^3 + x^2 = x^2 + x$$
$$x^3 - x = 0$$
$$x(x^2 - 1) = 0$$
$$x(x-1)(x+1) = 0$$
$$x = 0 \quad x - 1 = 0 \quad x + 1 = 0$$
$$x = 1 \quad x = -1$$

Upper/Lower

$-\frac{1}{2} \in (-1,0)$

$y = x^3 + x^2$	$y = x^2 + x$
$y = (-\frac{1}{2})^3 + (-\frac{1}{2})^2$	$y = (-\frac{1}{2})^2 + (-\frac{1}{2})$
$y = -\frac{1}{8} + \frac{1}{4}$	$y = \frac{1}{4} - \frac{2}{4}$
$y = -\frac{1}{8} + \frac{2}{8}$	$y = -\frac{1}{4}$
$y = \frac{1}{8}$	
upper	Lower

Upper/Lower

$\frac{1}{2} \in (0,1)$

$y = x^3 + x^2$	$y = x^2 + x$
$y = (\frac{1}{2})^3 + (\frac{1}{2})^2$	$y = (\frac{1}{2})^2 + (\frac{1}{2})$
$y = \frac{1}{8} + \frac{1}{4}$	$y = \frac{1}{4} + \frac{2}{4}$
$y = \frac{1}{8} + \frac{2}{8}$	$y = \frac{3}{4}$
$y = \frac{3}{8}$	
Lower	upper

$$A = \int_{-1}^0 [(x^3 + x^2) - (x^2 + x)]\,dx + \int_0^1 [(x^2 + x) - (x^3 + x^2)]\,dx$$
$$= \int_{-1}^0 [x^3 - x]\,dx + \int_0^1 [x - x^3]\,dx$$
$$= \left[\tfrac{1}{4}x^4 - \tfrac{1}{2}x^2\right]\Big|_{-1}^0 + \left[\tfrac{1}{2}x^2 - \tfrac{1}{4}x^4\right]\Big|_0^1$$
$$= [\tfrac{1}{4}(0)^4 - \tfrac{1}{2}(0)^2] - [\tfrac{1}{4}(-1)^4 - \tfrac{1}{2}(-1)^2] + [\tfrac{1}{2}(1)^2 - \tfrac{1}{4}(1)^4] - [\tfrac{1}{2}(0)^2 - \tfrac{1}{4}(0)^4]$$
$$= [0] - [\tfrac{1}{4}(1) - \tfrac{1}{2}(1)] + [\tfrac{1}{2}(1) - \tfrac{1}{4}(1)] - [0]$$
$$= -[\tfrac{1}{4} - \tfrac{2}{4}] + [\tfrac{2}{4} - \tfrac{1}{4}]$$
$$= -[-\tfrac{1}{4}] + [\tfrac{1}{4}]$$
$$= \tfrac{1}{4} + \tfrac{1}{4}$$
$$A = \tfrac{1}{2}\text{ un}^2$$

#1: $\int_4^{10} \left[\frac{\tan(x)\cos(x)}{\sin(x)}\right] dx = 6\, un^2$

#2: $\int_0^4 x dx = 8un.^2$

#3: $\int_0^1 e^x dx = (e-1)un.^2$

#4: $\int_0^9 (9-3\sqrt{x})dx = 27un.^2$

#5: $\int_e^{e^3} \frac{1}{x} dx = 2un.^2$

#6: $\int_0^1 (6x^2 - 4e^{2x})dx = (4-2e^2)un.^2$

#7:

$$\int_1^2 \frac{(x+1)^2}{x^2} dx = (1.5 + 2\ln 2)un.^2 \approx 2.89un.^2$$

#8: The total weight gain of a child from age 1 to age 9 is about 68 pounds.

#9: The total cost of the making the first one hundred units is about $505.70.

#10: The average number of marriages from the year 1995 to the year 2005 is about 2.4.9 million. (2,419,000 marriages).

#11: An indefinite integral results in the original function only (the arbitrary constant attached to the answer).
A definite integral results in a specific number because the original function is then evaluated at the upper and lower limits and then subtracted. The result is the area under the curve between the two limits. (The arbitrary constant is left out because it will cancel anyhow due to the subtraction in the formula.)

#12: "The area under the curve x^2 between x = 3 and x = 7."

#13: $2un.^2$

#14: $\frac{1}{2}un.^2$