Limits & Continuity
1.3A – One Sided Limits & Graphing

A: Translate into words:

#1)

$x \to 10^-$

x approaches 10 from the left.

$x \to -10$

x approaches negative 10 from both sides.

$x \to -10^-$

x approaches negative 10 from the left.

B: Find each limit by substitution.

#2) $f(x) = \begin{cases} x+6 & \text{if } x \le 2 \\ 2x-5 & \text{if } x > 2 \end{cases}$

$\lim_{x\to 2^-} (x+6) = (2)+6 = 8$

$\lim_{x\to 2^+} (2x-5) = 2(2)-5 = 4-5 = -1$

a. $\lim_{x\to 2^-} f(x) = 8$

b. $\lim_{x\to 2^+} f(x) = -1$

c. $\lim_{x\to 2} f(x) =$ dne

#3) $f(x) = \begin{cases} -3x+6 & \text{if } x \le 0 \\ 2x+6 & \text{if } x > 0 \end{cases}$

$\lim_{x\to 0^-} (-3x+6) = -3(0)+6 = 6$

$\lim_{x\to 0^+} (2x+6) = 2(0)+6 = 6$

a. $\lim_{x\to 0^-} f(x) = 6$

b. $\lim_{x\to 0^+} f(x) = 6$

c. $\lim_{x\to 0} f(x) = 6$

B: Find each limit.

#4) $f(x) = |x|$

$f(x) = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$

a. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} -x = -(0) = 0$

b. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = (0) = 0$

c. $\lim_{x\to 0} f(x) = 0$

#5) $f(x) = \frac{|x|}{x}$

If $x < 0$, then $f(x) = \frac{|-x|}{-x} = \frac{x}{-x}$, $f(x) = -1$, $\therefore f(x) = -1$ if $x < 0$

If $x > 0$, then $f(x) = \frac{|+x|}{+x} = \frac{+x}{+x}$, $f(x) = 1$, $\therefore f(x) = 1$ if $x > 0$

$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$

a. $\lim_{x\to 0^-} f(x) = -1$

b. $\lim_{x\to 0^+} f(x) = 1$

c. $\lim_{x\to 0} f(x) =$ dne

#6) $f(x) = \frac{-x}{|x|}$

If $x < 0$, then $f(x) = \frac{-(-x)}{|-x|} = \frac{x}{x}$, $f(x) = 1$, $\therefore f(x) = 1$ if $x < 0$

If $x > 0$, then $f(x) = \frac{-(+x)}{|+x|} = \frac{-x}{x}$, $f(x) = -1$, $\therefore f(x) = -1$ if $x > 0$

$f(x) = \begin{cases} 1 & \text{if } x < 0 \\ -1 & \text{if } x > 0 \end{cases}$

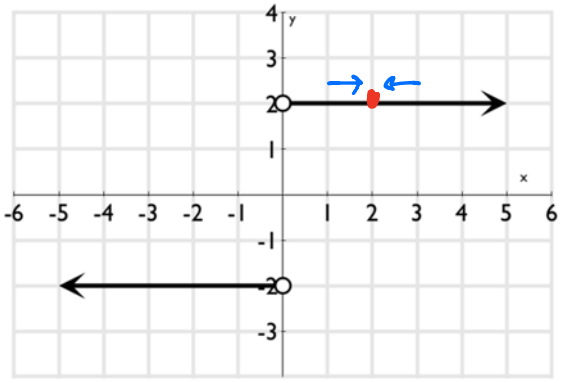
a. $\lim_{x\to 0^-} f(x) = 1$

b. $\lim_{x\to 0^+} f(x) = -1$

c. $\lim_{x\to 0} f(x) =$ dne

C: Find each limit.

#7)

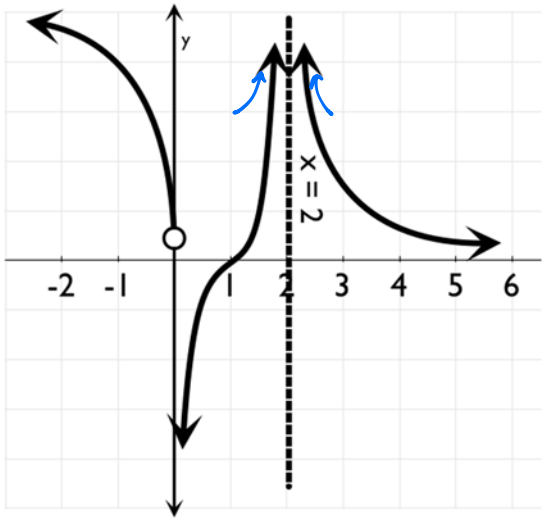


a. $\lim_{x\to 2^-} f(x) = 2$

b. $\lim_{x\to 2^+} f(x) = 2$

c. $\lim_{x\to 2} f(x) = 2$

#8)

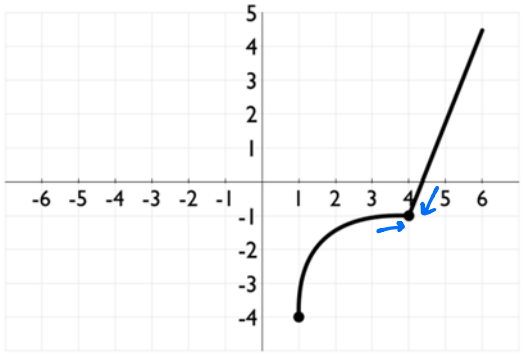


a. $\lim_{x\to 2^-} f(x) = \infty$, dne

b. $\lim_{x\to 2^+} f(x) = \infty$, dne

c. $\lim_{x\to 2} f(x) = \infty$, dne

#9)

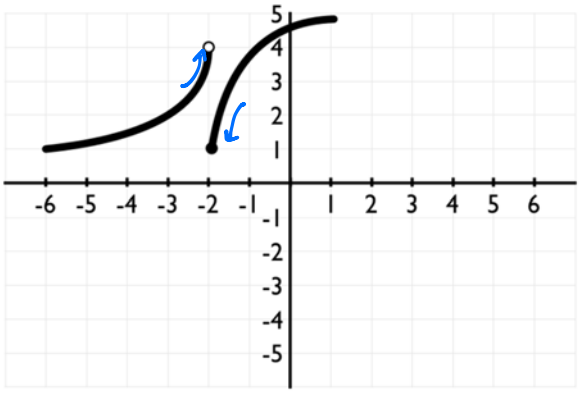


a. $\lim_{x\to 4^-} f(x) = -1$

b. $\lim_{x\to 4^+} f(x) = -1$

c. $\lim_{x\to 4} f(x) = -1$

#10)



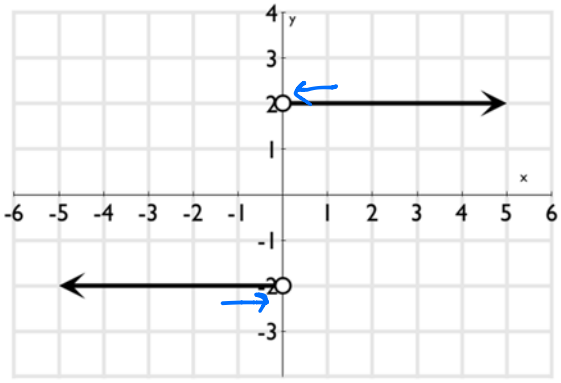
a. $\lim_{x\to -2^-} f(x) = 4$

b. $\lim_{x\to -2^+} f(x) = 1$

c. $\lim_{x\to -2} f(x) = \text{dne}$

C: Find each limit.

#11)

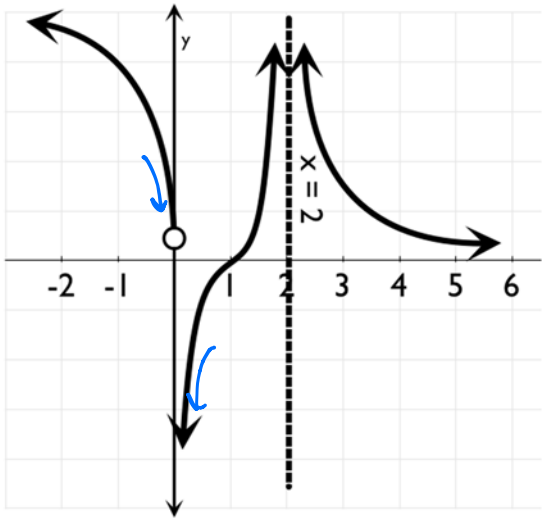


a. $\lim_{x\to 0^-} f(x) = -2$

b. $\lim_{x\to 0^+} f(x) = 2$

c. $\lim_{x\to 0} f(x) = \text{dne}$

#12)

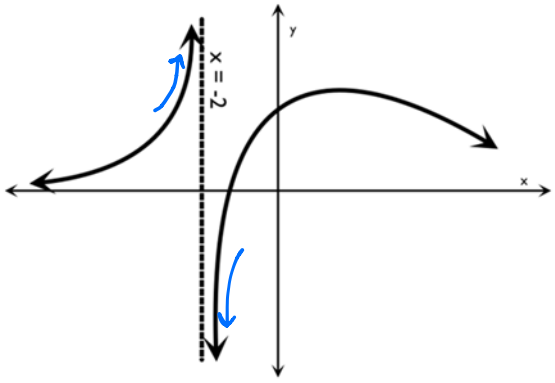


a. $\lim_{x\to 0^-} f(x) = \frac{1}{2}$

b. $\lim_{x\to 0^+} f(x) = -\infty, \text{dne}$

c. $\lim_{x\to 0} f(x) = \text{dne}$

#13)

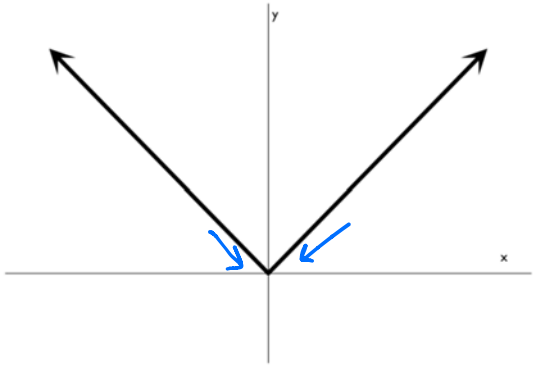


a. $\lim_{x\to -2^-} f(x) = \infty, \text{dne}$

b. $\lim_{x\to -2^+} f(x) = -\infty, \text{dne}$

c. $\lim_{x\to -2} f(x) = \text{dne}$

#14)



a. $\lim_{x\to 0^-} f(x) = 0$

b. $\lim_{x\to 0^+} f(x) = 0$

c. $\lim_{x\to 0} f(x) = 0$

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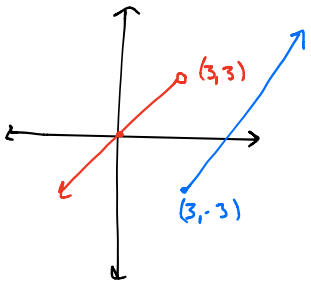
D: Draw each graph by hand. Find the limit as x approaches 3 from the left and from the right. Find the two sided limit.

#15) $f(x) = \begin{cases} x & if\ x < 3 \\ x - 6 & if\ x \geq 3 \end{cases}$

x	x
3	3
0	0

x	x − 6
3	−3
6	0



$\lim_{x \to 3^-} f(x) = 3$

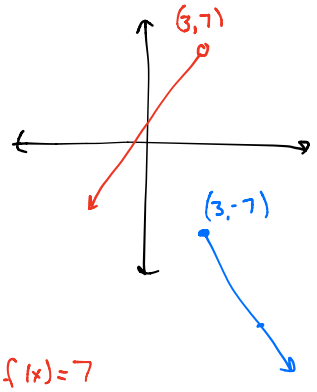
$\lim_{x \to 3^+} f(x) = -3$

$\lim_{x \to 3} f(x) = dne$

#16) $f(x) = \begin{cases} 2x + 1 & if\ x < 3 \\ -2x - 1 & if\ x \geq 3 \end{cases}$

x	2x+1
3	7
0	1

x	−2x−1
3	−7
6	−13



$\lim_{x \to 3^-} f(x) = 7$

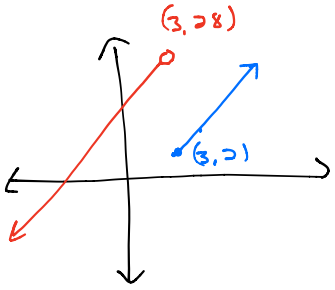
$\lim_{x \to 3^+} f(x) = -7$

$\lim_{x \to 3} f(x) = dne$

#17) $f(x) = \begin{cases} \frac{1}{3}x + 27 & if\ x < 3 \\ x - 1 & if\ x \geq 3 \end{cases}$

x	$\frac{1}{3}x+27$
3	28
0	27

x	x−1
3	2
6	5



$\lim_{x \to 3^-} f(x) = 28$

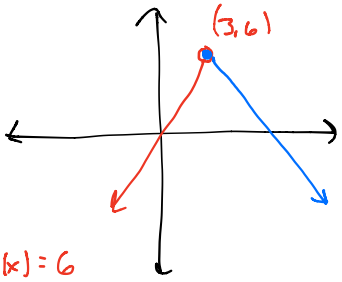
$\lim_{x \to 3^+} f(x) = 2$

$\lim_{x \to 3} f(x) = dne$

#18) $f(x) = \begin{cases} 2x & if\ x < 3 \\ -2x + 12 & if\ x \geq 3 \end{cases}$

x	2x
3	6
0	0

x	−2x+12
3	6
6	0



$\lim_{x \to 3^-} f(x) = 6$

$\lim_{x \to 3^+} f(x) = 6$

$\lim_{x \to 3} f(x) = 6$