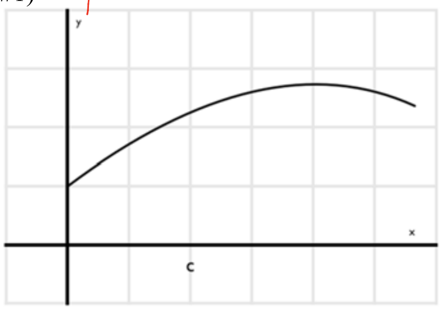
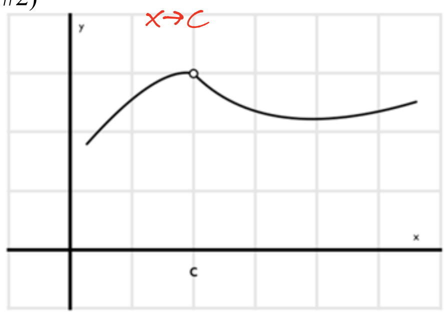
Limits & Continuity
1.4A – Continuity

A: Determine whether each function is continuous at c. If discontinuous, state why.

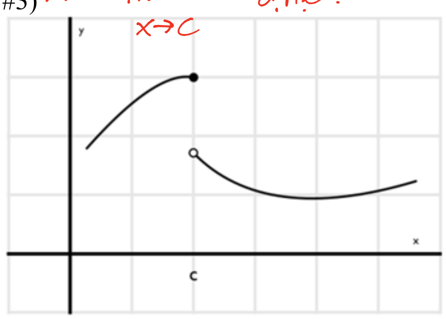
#1) Yes



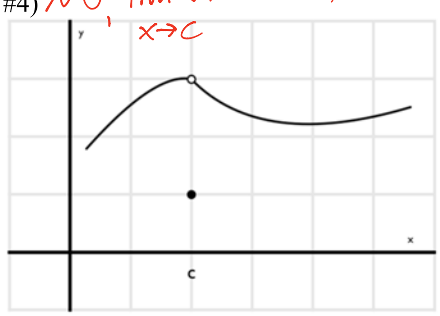
#2) No, $\lim_{x \to c} f(x) \neq f(c)$



#3) No $\lim_{x \to c} f(x) \neq$ d.n.e.



#4) NO, $\lim_{x \to c} f(x) \neq f(c)$



B: Determine whether each function is continuous. If discontinuous, state where it is discontinuous. (You've graphed some of these functions on previous homework.)

#5) $f(x) = 6x + 8$

Continuous

#6) $f(x) = \frac{x+2}{x-2}$

VA

$\boxed{\begin{array}{c} x-2=0 \\ x=2 \end{array}}$

Discontinuous @ $x=2$

#7) $f(x) = \frac{1}{x^2+29x+28}$

VA

$x^2+29x+28=0$

$(x+1)(x+28)=0$

$x+1=0 \quad x+28=0$

$x=-1 \quad x=-28$

Discontinuous @ $x=-28$ and -1

#8) $f(x) = \begin{cases} x & if\ x < 0 \\ x-6 & if\ x \geq 0 \end{cases}$

$f(x)=x$

$\boxed{f(0)=0}$

$f(x)=x-6$

$\boxed{\begin{array}{c} f(0)=(0)-6 \\ f(0)=-6 \end{array}}$

Discontinuous @ $x=0$

#9) $f(x) = \begin{cases} 2x+1 & \text{if } x < 2 \\ -2x-1 & \text{if } x \geq 2 \end{cases}$

$f(x) = 2x+1$

$f(2) = 2(2)+1$
$= 4+1$
$f(2) = 5$

$f(x) = -2x-1$

$f(2) = -2(2)-1$
$= -4-1$
$f(2) = -5$

Discontinuous @ $x = 2$

#10) $f(x) = \begin{cases} \frac{1}{3}x+5 & \text{if } x < 9 \\ x-1 & \text{if } x \geq 9 \end{cases}$

$f(x) = \frac{1}{3}x+5$

$f(9) = \frac{1}{3}(9)+5$
$= 3+5$
$f(9) = 8$

$f(x) = x-1$

$f(9) = (9)-1$
$f(9) = 8$

Continuous

#11) $f(x) = \begin{cases} 2x & \text{if } x < 3 \\ -2x+12 & \text{if } x \geq 3 \end{cases}$

$f(x) = 2x$

$f(3) = 2(3)$
$f(3) = 6$

$f(x) = -2x+12$

$f(3) = -2(3)+12$
$= -6+12$
$f(3) = 6$

Continuous

C: Decide if each statement is true or false. If false give a counterexample. (A counterexample makes the hypothesis true and the conclusion false.)

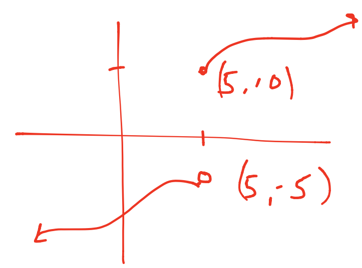
#12) If $\lim_{x\to5} f(x) = 10$, then $\lim_{x\to5^+} f(x) = 10$

True

#13) If $\lim_{x\to5^+} f(x) = 10$, then $\lim_{x\to5} f(x) = 10$

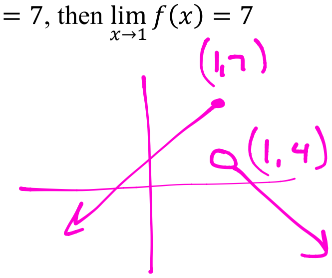
False

Counterexample



#14) If $f(1) = 7$, then $\lim_{x\to1} f(x) = 7$

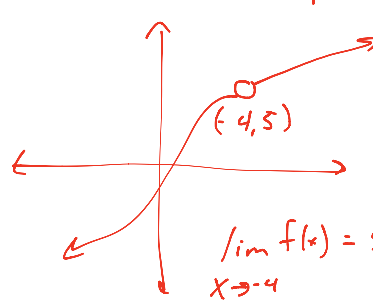
FALSE



#15) If $f(-4)$ is not defined, then $\lim_{x\to-4} f(x)$ does not exist.

False

Counterexample



$\lim_{x\to-4} f(x) = 5$