10.1A – Integration by Parts

A: Find the following integrals WITHOUT using integration by parts. Use formulas 1 through 7

#1) $\int e^{5x}dx$

$= \frac{1}{5}e^{5x} + C$

#2) $\int (x+7)dx$

$= \frac{1}{2}x^2 + 7x + C$

#3) $\int \sqrt[3]{x}\,dx$

$= \frac{3}{4}x^{4/3} + C$

#4) $\int (3x-9)^5 dx$

$u = 3x-9$
$\frac{du}{dx} = 3$
$du = 3dx$
$\frac{du}{3} = dx$

$= \int u^5 \frac{du}{3}$

$= \frac{1}{18}u^6 + C$

$= \frac{1}{18}(3x-9)^6 + C$

#5) $\int \frac{1}{(2x-6)^3}dx$

$u = 2x-6$
$\frac{du}{dx} = 2$
$du = 2dx$
$\frac{du}{2} = dx$

$= \int \frac{1}{u^3}\frac{du}{2}$

$= \frac{1}{2}\int u^{-3}du$

$= -\frac{1}{4}u^{-2} + C$

$= \frac{-1}{4(2x-6)^2} + C$

#6) $\int \frac{1}{x}dx$

$= \ln|x| + C$

B: Use integration by parts to find each integral.

#7) $\int xe^{2x}dx$

$u = x$ | $dv = e^{2x}dx$
$\frac{du}{dx} = 1$ | $\int dv = \int e^{2x}dx$
$du = dx$ | $v = \frac{1}{2}e^{2x}$

$\int u\,dv = uv - \int v\,du$

$\int x e^{2x} dv = x\left(\frac{1}{2}e^{2x}\right) - \int \frac{1}{2}e^{2x}dx$

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

#8) $\int x^5 \ln x\,dx$

$u = \ln(x)$ | $dv = x^5 dx$
$\frac{du}{dx} = \frac{1}{x}$ | $\int dv = \int x^5 dx$
$du = \frac{1}{x}dx$ | $v = \frac{1}{6}x^6$

$\int u\,dv = uv - \int v\,du$

$\int \ln(x)\cdot x^5 dx = \ln(x)\left(\frac{1}{6}x^6\right) - \int \frac{1}{6}x^6\left(\frac{1}{x}dx\right)$

$= \frac{1}{6}x^6\ln(x) - \frac{1}{6}\int x^5 dx$

$= \frac{1}{6}x^6\ln(x) - \frac{1}{36}x^6 + C$

#9) $\int (5x+2)e^x\,dx$

$u = 5x+2$ $dv = e^x dx$

$\frac{du}{dx} = 5$ $\int dv = \int e^x dx$

$du = 5dx$ $v = e^x$

$\int u\,dv = uv - \int v\,du$

$\int (5x+2)e^x dx = (5x+2)(e^x) - \int e^x 5dx$

$= e^x(5x+2) - 5e^x + C$

#10) $\int \sqrt{x}\ln x\,dx$

$u = \ln(x)$ $dv = x^{\frac{1}{2}}dx$

$\frac{du}{dx} = \frac{1}{x}$ $\int dv = \int x^{\frac{1}{2}}dx$

$du = \frac{1}{x}dx$ $v = \frac{2}{3}x^{\frac{3}{2}}$

$\int u\,dv = uv - \int v\,du$

$\int \ln(x)\cdot\sqrt{x}\,dx = \ln(x)\cdot\frac{2}{3}\sqrt{x^3} - \int \frac{2}{3}x^{\frac{3}{2}}(x^{-1}dx)$

$= \frac{2}{3}\sqrt{x^3}\ln(x) - \frac{2}{3}\int x^{\frac{1}{2}}dx$

$= \frac{2}{3}\sqrt{x^3}\ln(x) - \frac{2}{3}\cdot\frac{2}{3}x^{\frac{3}{2}} + C$

$= \frac{2}{3}\sqrt{x^3}\ln(x) - \frac{4}{9}\sqrt{x^3} + C$

#11) $\int \frac{\ln t}{t^2}dt$

$u = \ln(t)$ $dv = t^{-2}dt$

$\frac{du}{dt} = \frac{1}{t}$ $\int dv = \int t^{-2}dt$

$du = \frac{1}{t}dt$ $v = -t^{-1}$

$\int u\,dv = uv - \int v\,du$

$\int \ln(t)\cdot t^{-2}dt = \ln(t)\cdot(-t^{-1}) - \int -t^{-1}\cdot\frac{1}{t}dt$

$= \frac{\ln(t)}{-t} + \int t^{-2}dt$

$= \frac{\ln(t)}{-t} - t^{-1} + C$

$= \frac{\ln(t)}{-t} - \frac{1}{t} + C$

#12) $\int t\,e^{-0.5t}dt$

$u = t$ $dv = e^{-0.5t}dt$

$\frac{du}{dt} = 1$ $\int dv = \int e^{-0.5t}dt$

$du = dt$ $v = -2e^{-0.5t}$

$\int u\,dv = uv - \int v\,du$

$\int t\,e^{-0.5t}dt = t(-2e^{-0.5t}) - \int -2e^{-0.5t}dt$

$= -2te^{-0.5t} - 4e^{-0.5t} + C$

#1) $\frac{1}{5}e^{5x} + c$

#2) $\frac{1}{2}x^2 + 7x + c$

#3) $\frac{3}{4}x^{4/3} + c$

#4) $\frac{1}{18}(3x-9)^6 + c$

#5) $\frac{-1}{4(2x-6)^2} + c$

#6) $\ln|x| + c$

#7) $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$

#8) $\frac{1}{6}x^6\ln x - \frac{1}{36}x^6 + c$

#9) $(5x+2)e^x - 5e^x + c$

#10) $\frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + c$

#11) $\frac{-1}{t}\ln t - \frac{1}{t} + c$

#12) $-2te^{-0.5t} - 4e^{-0.5t} + c$