Advanced Integration

10.1B – Integration by Parts

Use integration by parts to find each integral.

#1) $\int (x-3)(3x+4)^5\,dx$

$u = x-3$, $\frac{du}{dx} = 1$, $du = dx$

$dv = (3x+4)^5\,dx$

$\int dv = \int (3x+4)^5\,dx$

$v = \int p^5 \frac{dp}{3}$

$v = \frac{1}{18}p^6$

$v = \frac{1}{18}(3x+4)^6$

$p = 3x+4$, $\frac{dp}{dx} = 3$, $dp = 3dx$, $\frac{dp}{3} = dx$

$$\int u\,dv = uv - \int v\,du$$

$$= (x-3)\left(\tfrac{1}{18}(3x+4)^6\right) - \int \tfrac{1}{18}(3x+4)^6\,dx$$

$$= \tfrac{1}{18}(x-3)(3x+4)^6 - \tfrac{1}{18}\int p^6 \tfrac{dp}{3}$$

$$= \tfrac{1}{18}(x-3)(3x+4)^6 - \tfrac{1}{378}p^7 + C$$

$$= \tfrac{1}{18}(x-3)(3x+4)^6 - \tfrac{1}{378}(3x+4)^7 + C$$

#2) $\int t(2t+1)^4 dt$

$u = t$, $\frac{du}{dt} = 1$, $du = dt$

$dv = (2t+1)^4 dt$

$\int dv = \int (2t+1)^4 dt$

$v = \int p^4 \frac{dp}{2}$

$v = \frac{1}{10}p^5$

$v = \frac{1}{10}(2t+1)^5$

$p = 2t+1$, $\frac{dp}{dt} = 2$, $dp = 2dt$, $\frac{dp}{2} = dt$

$$\int u\,dv = uv - \int v\,du$$

$$= t\left(\tfrac{1}{10}(2t+1)^5\right) - \int \tfrac{1}{10}(2t+1)^5 dt$$

$$= \tfrac{t}{10}(2t+1)^5 - \tfrac{1}{10}\int p^5 \tfrac{dp}{2}$$

$$= \tfrac{t}{10}(2t+1)^5 - \tfrac{1}{120}p^6 + C$$

$$= \tfrac{t}{10}(2t+1)^5 - \tfrac{1}{120}(2t+1)^6 + C$$

#3) $\int \frac{x}{e^{2x}}\,dx$

$u = x$, $\frac{du}{dx} = 1$, $du = dx$

$dv = e^{-2x}\,dx$

$\int dv = \int e^{-2x}\,dx$

$v = -\frac{1}{2}e^{-2x}$

$v = -\frac{1}{2e^{2x}}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x e^{-2x}\,dx = x\left(\tfrac{-1}{2e^{2x}}\right) - \int -\tfrac{1}{2}e^{-2x}\,dx$$

$$= \tfrac{-x}{2e^{2x}} - \tfrac{1}{4}e^{-2x} + C$$

$$= \tfrac{-x}{2e^{2x}} - \tfrac{1}{4e^{2x}} + C$$

#4) $\int \frac{x}{\sqrt{x+1}}\,dx$

$u = x$, $\frac{du}{dx} = 1$, $du = dx$

$dv = (x+1)^{-\frac{1}{2}}\,dx$

$\int dv = \int (x+1)^{-\frac{1}{2}}\,dx$

$v = \int p^{-\frac{1}{2}}\,dp$

$v = 2p^{\frac{1}{2}}$

$v = 2\sqrt{x+1}$

$p = x+1$, $dp = dx$

$$\int u\,dv = uv - \int v\,du$$

$$\int x(x+1)^{-\frac{1}{2}}dx = x\left(2\sqrt{x+1}\right) - \int 2(x+1)^{\frac{1}{2}}\,dx$$

$$= 2x\sqrt{x+1} - 2\int p^{\frac{1}{2}}\,dp$$

$$= 2x\sqrt{x+1} - \tfrac{4}{3}p^{\frac{3}{2}} + C$$

$$= 2x\sqrt{x+1} - \tfrac{4}{3}\sqrt{(x+1)^3} + C$$

#5) $\int xe^{ax}\,dx \qquad (a \neq 0)$

$u = x \qquad dv = e^{ax}dx$

$\frac{du}{dx} = 1 \qquad \int dv = \int e^{ax}dx$

$du = dx \qquad v = \frac{1}{a}e^{ax}$

$\int u\,dv = uv - \int v\,du$

$\int x e^{ax}dx = x\left(\frac{1}{a}e^{ax}\right) - \int \frac{1}{a}e^{ax}dx$

$= \frac{x}{a}e^{ax} - \frac{e^{ax}}{a^2} + C$

#6) $\int x^n \ln(ax)\,dx \qquad (a \neq 0, n \neq 1)$

$u = \ln(ax) \qquad dv = x^n dx$

$\frac{du}{dx} = \frac{a}{ax} \qquad \int dv = \int x^n dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{n+1}x^{n+1}$

$\int u\,dv = uv - \int v\,du$

$\int \ln(ax)\,x^n dx = \ln(ax)\,\frac{1}{n+1}x^{n+1} - \int \frac{1}{n+1}x^{n+1}\cdot\frac{1}{x}dx$

$= \frac{1}{n+1}x^{n+1}\ln(ax) - \frac{1}{n+1}\int x^n\,dx$

$= \frac{1}{n+1}x^{n+1}\ln(ax) - \frac{1}{(n+1)^2}x^{n+1} + C$

#7) $\int \ln x\,dx$

$u = \ln x \qquad dv = dx$

$\frac{du}{dx} = \frac{1}{x} \qquad \int dv = \int dx$

$du = \frac{1}{x}dx \qquad v = x$

$\int u\,dv = uv - \int v\,du$

$\int \ln x\,dx = \ln x \cdot x - \int x\left(\frac{1}{x}dx\right)$

$= x\ln x - \int dx$

$= x\ln x - x + C$

#8) $\int x^3 e^{x^2}dx = \int x^2 \cdot x e^{x^2}dx$

$u = x^2 \qquad dv = xe^{x^2}dx$

$\frac{du}{dx} = 2x \qquad \int dv = \int xe^{x^2}dx$

$du = 2x\,dx \qquad v = \int xe^p\frac{dp}{2x}$

$v = \frac{1}{2}\int e^p dp$

$v = \frac{1}{2}e^p$

$v = \frac{1}{2}e^{x^2}$

$p = x^2$

$\frac{dp}{dx} = 2x$

$dp = 2x\,dx$

$\frac{dp}{2x} = dx$

$\int u\,dv = uv - \int v\,du$

$= x^2\left(\frac{1}{2}e^{x^2}\right) - \int \frac{1}{2}e^{x^2}\cdot 2x\,dx$

$= \frac{1}{2}x^2e^{x^2} - \int xe^{x^2}dx$

$= \frac{1}{2}x^2e^{x^2} - \int xe^p\frac{dp}{2x}$

$= \frac{1}{2}x^2e^{x^2} - \frac{1}{2}\int e^p\,dp$

$= \frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^p + C$

$= \frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + C$

#1) $\frac{1}{18}(x-3)(3x+4)^6 - \frac{1}{378}(3x+4)^7 + c$

#2) $\frac{1}{10}t(2t+1)^5 - \frac{1}{120}(2t+1)^6 + c$

#3) $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c$

#4) $2x\sqrt{x+1} - \frac{4}{3}\sqrt{(x+1)^3} + c$

#5) $\frac{1}{a}xe^{ax} - \frac{1}{a^2}e^{ax} + c$

#6) $\frac{1}{n+1}x^{n+1}\ln(ax) - \frac{1}{(n+1)^2}x^{n+1} + c$

#7) $x\ln x - x + c$

#8) $\frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + c$