Find each integral by integration by parts or a substitution. Choose wisely.

#1) $\int xe^{x^2}dx = \int xe^p \frac{dp}{2x}$

$= \frac{1}{2}\int e^p dp$

$= \frac{1}{2}e^p$

$= \frac{1}{2}e^{x^2}+C$

$p = x^2$
$\frac{dp}{dx} = 2x$
$dp = 2x\,dx$
$\frac{dp}{2x} = dx$

#2) $\int \frac{(\ln x)^3}{x}dx = \int \frac{p^3}{x}(x\,dp)$

$p = \ln x$
$\frac{dp}{dx} = \frac{1}{x}$
$dp = \frac{1}{x}dx$
$x\,dp = dx$

$= \int p^3 dp$

$= \frac{1}{4}p^4 + C$

$= \frac{1}{4}(\ln x)^4 + C$

#3) $\int x^2 \ln(2x)\,dx$

$u = \ln(2x)$ $\quad dv = \int x^2 dx$
$\frac{du}{dx} = \frac{2}{2x}$ $\quad \int dv = \int x^2 dx$
$du = \frac{1}{x}dx$ $\quad v = \frac{1}{3}x^3$

$\int u\,dv = uv - \int v\,du$

$= \ln(2x)\cdot\frac{1}{3}x^3 - \int \frac{1}{3}x^3 \frac{1}{x}dx$

$= \frac{1}{3}x^3\ln(2x) - \frac{1}{3}\int x^2 dx$

$= \frac{1}{3}x^3\ln(2x) - \frac{1}{9}x^3 + C$

#4) $\int \frac{e^x}{e^x+4}dx = \int \frac{e^x}{p}\left(\frac{dp}{e^x}\right)$

$p = e^x + 4$
$\frac{dp}{dx} = e^x$
$dp = e^x dx$
$\frac{dp}{e^x} = dx$

$= \int \frac{1}{p}dp$

$= \ln|p| + C$

$= \ln|e^x+4| + C$

Evaluate each definite integral using integration by parts. Exact answers only.

#5) $\int_0^2 xe^x dx$

$u = x$ $\quad dv = e^x dx$
$du = dx$ $\quad v = e^x$

$\int u\,dv = uv - \int v\,du$

$\int xe^x dx = xe^x - \int e^x dx$

$= xe^x - e^x + C$

$\int_0^2 xe^x dx = \left[xe^x - e^x\right]\Big|_0^2$

$= \left[2e^2 - e^2\right] - \left[0e^0 - e^0\right]$

$= \left[e^2\right] - \left[0 - 1\right]$

$= e^2 + 1$

#6) $\int_1^3 x^2 \ln x\,dx$

$u = \ln x$ $\quad dv = x^2 dx$
$du = \frac{1}{x}dx$ $\quad v = \frac{1}{3}x^3$

$\int u\,dv = uv - \int v\,du$

$\int \ln x\,(x^2 dx) = \ln x\left(\frac{1}{3}x^3\right) - \int \frac{1}{3}x^3\left(\frac{1}{x}dx\right)$

$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$

$\int_1^3 x^2 \ln x\,dx = \left(\frac{1}{3}x^3\ln x - \frac{1}{9}x^3\right)\Big|_1^3$

$= \left[\frac{1}{3}(3)^3\ln(3) - \frac{1}{9}(3)^3\right] - \left[\frac{1}{3}(1)^3\ln(1) - \frac{1}{9}(1)^3\right]$

$= \left[\frac{1}{3}(27)\ln 3 - \frac{1}{9}(27)\right] - \left[\frac{1}{3}(1)(0) - \frac{1}{9}(1)\right]$

$= \left[9\ln 3 - \frac{27}{9}\right] - \left[0 - \frac{1}{9}\right]$

$= 9\ln 3 - \frac{26}{9}$

#7) $\int_0^2 x(x-2)^4 dx$

$u = x \quad dv = (x-2)^4 dx$
$v = \int p^4 dp$ $\quad p = x-2$, $dp = dx$
$du = dx \quad v = \frac{1}{5}(x-2)^5$

$\int u\,dv = uv - \int v\,du$

$\int x(x-2)^4 dx = x\left(\frac{1}{5}(x-2)^5\right) - \int \frac{1}{5}(x-2)^5 dx$

$= \frac{1}{5}x(x-2)^5 - \frac{1}{5}\int p^5 dx$

$= \frac{1}{5}x(x-2)^5 - \frac{1}{30}p^6 + C$

$= \frac{1}{5}x(x-2)^5 - \frac{1}{30}(x-2)^6 + C$

$\int_0^2 x(x-2)^4 dx = \left[\frac{1}{5}x(x-2)^5 - \frac{1}{30}(x-2)^6\right]\Big|_0^2$

$= \left[\frac{1}{5}(2)(2-2)^5 - \frac{1}{30}(2-2)^6\right] - \left[\frac{1}{5}(0)(0-2)^5 - \frac{1}{30}(0-2)^6\right]$

$= \left[\frac{2}{5}(0)^5 - \frac{1}{30}(0)^6\right] - \left[\frac{1}{5}(0)(-2)^5 - \frac{1}{30}(-2)^6\right]$

$= [\ 0\ ] + \left[\frac{64}{30}\right]$

$= \frac{32}{15}$

#8) $\int_0^{\ln 4} xe^x dx$

From problem #5

$\int xe^x dx = xe^x - e^x + C$

$\int_0^{\ln 4} xe^x dx = \left[xe^x - e^x\right]\Big|_0^{\ln 4}$

$= \left[(\ln 4)e^{\ln 4} - e^{\ln 4}\right] - \left[0 e^0 - e^0\right]$

$= [4\ln 4 - 4] - [-1]$

$= 4\ln 4 - 3$

Find each integral by repeating integration by parts.

#9) $\int x^2 e^{-x} dx$

$u = x^2 \quad dv = e^{-x} dx$
$du = 2x\,dx \quad v = -e^{-x}$

$\int u\,dv = uv - \int v\,du$

$\int x^2 e^{-x} dx = x^2(-e^{-x}) - \int -e^{-x}(2x\,dx)$

$= -x^2 e^{-x} + 2\int x e^{-x} dx$

$u = x \quad dv = e^{-x} dx$
$du = dx \quad v = -e^{-x}$

$= -x^2 e^{-x} + 2\left[x(-e^{-x}) - \int -e^{-x} dx\right]$

$= -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + C$

#1)	$\frac{1}{2}e^{x^2} + c$ (sub)		#6)	$9\ln 3 - \frac{26}{9}$
#2)	$\frac{1}{4}(\ln x)^4 + c$ (sub)		#7)	$\frac{32}{15}$
#3)	$\frac{1}{3}x^3\ln(2x) - \frac{1}{9}x^3 + c$ (by parts)		#8)	$-3 + 4\ln 4$
#4)	$\ln(e^x + 4) + c$ (sub)		#9)	$-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + c$
#5)	$e^2 + 1$			