Advanced Integration
10.2A – Complex Integration Formulas

State the number of the complex integration formula you would use to find each integral. Also, state the values you would use for the substitution.

#1) $\int \frac{1}{x^2(3x-1)}dx$

Formula 12: $\int \frac{1}{z^2(az+b)}dz$

$z^2 = x^2$ $\quad a = 3$

$z = x$ $\quad b = -1$

$dz = dx$

#2) $\int \frac{1}{x\sqrt{-x+2}}dx$

Formula 14: $\int \frac{1}{z\sqrt{az+b}}dz$

$z = x$ $\quad a = -1$

$dz = dx$ $\quad b = 2$

#3) $\int \frac{x}{1-x}dx = \int \frac{x}{-x+1}dx$

Formula 9: $\int \frac{z}{az+b}dz$

$z = x$ $\quad a = -1$

$dz = dx$ $\quad b = 1$

#4) $\int \frac{x}{4x-5}dx$

Formula 9: $\int \frac{z}{az+b}dz$

$z = x$ $\quad a = 4$

$dz = dx$ $\quad b = -5$

#5) $\int \frac{x}{\sqrt{-8x+1}}dx$

Formula 13: $\int \frac{z}{\sqrt{az+b}}dz$

$z = x$ $\quad a = -8$

$dz = dx$ $\quad b = 1$

#6) $\int \frac{1}{x\sqrt{1-2x}}dx = \int \frac{1}{x\sqrt{-2x+1}}dx$

Formula 14: $\int \frac{1}{z\sqrt{az+b}}dz$

$z = x$ $\quad a = -2$

$dz = dx$ $\quad b = 1$

Find each integral by using a complex integration formula.

#7) $\int \frac{1}{25-x^2}dx$

Use formula 16

$$\int \frac{1}{a^2-z^2}dz = \frac{1}{2a}\ln\left|\frac{a+z}{a-z}\right| + C$$

$z^2 = x^2$ $\quad a^2 = 25$

$z = x$ $\quad a = 5$

$dz = dx$

$$\int \frac{1}{25-x^2}dx = \frac{1}{2(5)}\ln\left|\frac{5+x}{5-x}\right| + C$$

$$= \frac{1}{10}\ln\left|\frac{5+x}{5-x}\right| + C$$

#8) $\int \frac{1}{x^2-81}dx$

Use formula 15

$$\int \frac{1}{z^2-a^2}dz = \frac{1}{2a}\ln\left|\frac{z-a}{z+a}\right| + C$$

$z^2 = x^2$ $\quad a^2 = 81$

$z = x$ $\quad a = 9$

$dz = dx$

$$\int \frac{1}{x^2-81}dx = \frac{1}{2(9)}\ln\left|\frac{x-9}{x+9}\right| + C$$

$$= \frac{1}{18}\ln\left|\frac{x-9}{x+9}\right| + C$$

#9) $\int \frac{1}{x^2(4x+7)}dx$

Use formula 12

$$\int \frac{1}{z^2(az+b)}dz = \frac{1}{-b}\left(\frac{1}{z} + \frac{a}{b}\ln\left|\frac{z}{az+b}\right|\right) + C$$

$z^2 = x^2$ $\quad a = 4$

$z = x$ $\quad b = 7$

$dz = dx$

$$\int \frac{1}{x^2(4x+7)}dx = \frac{1}{-7}\left(\frac{1}{x} + \frac{4}{7}\ln\left|\frac{x}{4x+7}\right|\right) + C$$

Advanced Integration
10.2A – Complex Integration Formulas

#10) $\int \frac{x}{x+9}dx$

Use formula 9

$\int \frac{z}{az+b}dz = \frac{z}{a} - \frac{b}{a^2}\ln|az+b| + C$

$z = x$	$a = 1$
$dz = dx$	$b = 9$

$\int \frac{x}{1\cdot x+9}dx = \frac{x}{1} - \frac{9}{(1)^2}\ln|1\cdot x+9| + C$

$= x - 9\ln|x+9| + C$

#11) $\int \frac{x}{1-x}dx$

Use formula 9

$= \int \frac{x}{-x+1}dx$

$\int \frac{z}{az+b}dz = \frac{z}{a} - \frac{b}{a^2}\ln|az+b| + C$

$z = x$	$a = -1$
$dz = dx$	$b = 1$

$\int \frac{x}{-1\cdot x+1}dx = \frac{x}{-1} - \frac{1}{(-1)^2}\ln|-1\cdot x+1| + C$

$= -x - \ln|-x+1| + C$

#12) $\int \frac{x}{\sqrt{2-x}}dx$ $= \int \frac{x}{\sqrt{-x+2}}dx$

Use formula 13

$\int \frac{z}{\sqrt{az+b}}dz = \frac{2az-4b}{3a^2}\sqrt{az+b} + C$

$z = x$	$a = -1$
$dz = dx$	$b = 2$

$\int \frac{x}{\sqrt{-1\cdot x+2}}dx = \frac{2(-1)x - 4(2)}{3(-1)^2}\sqrt{-1\cdot x+2} + C$

$= \frac{-2x-8}{3}\sqrt{-x+2} + C$

#13) $\int \frac{1}{(2x+5)(3x+1)}dx$

Formula 10: $\int \frac{1}{(az+b)(cz+d)}dz = \frac{1}{ad-bc}\ln\left|\frac{az+b}{cz+d}\right| + C$

$z = x$	$a = 2$	$c = 3$
$dz = dx$	$b = 5$	$d = 1$

$\int \frac{1}{(2\cdot x+5)(3x+1)}dx = \frac{1}{2(1)-5(3)}\ln\left|\frac{2x+5}{3x+1}\right| + C$

$= \frac{1}{-13}\ln\left|\frac{2x+5}{3x+1}\right| + C$

#14) $\int \sqrt{x^2-36}dx$

Formula 17: $\int \sqrt{z^2-a^2}\,dz = \frac{z}{2}\sqrt{z^2-a^2} - \frac{a^2}{2}\ln|z+\sqrt{z^2-a^2}| + C$

$z^2 = x^2$	$a^2 = 36$
$z = x$	$a = 6$
$dz = dx$	

$\int \sqrt{x^2-36}\,dx = \frac{x}{2}\sqrt{x^2-36} - \frac{36}{2}\ln|x+\sqrt{x^2-36}| + C$

$= \frac{x}{2}\sqrt{x^2-36} - 18\ln|x+\sqrt{x^2-36}| + C$

#15) $\int \frac{1}{x\sqrt{1-x^2}}dx$

Formula 20: $\int \frac{1}{z\sqrt{a^2-z^2}}dz = \frac{1}{-a}\ln\left|\frac{a+\sqrt{a^2-z^2}}{z}\right| + C$

$z^2 = x^2$	$a^2 = 1$
$z = x$	$a = 1$
$dz = dx$	

$\int \frac{1}{x\sqrt{1-z^2}}dz = \frac{1}{-1}\ln\left|\frac{1+\sqrt{1-x^2}}{x}\right| + C$

$= -\ln\left|\frac{1+\sqrt{1-x^2}}{x}\right| + C$