Advanced Integration
10.4 – Definite Complex Integrals

#1) Finding Total Sales from the Sales Rate

The sales rate of the Times Reporter is $\frac{x}{\sqrt{x+9}}$ thousand sales per week after x weeks. Find the total sales from week 7 to week 16.

$$\begin{pmatrix} \text{week } 7 \text{ to } 16 \\ \text{total} \end{pmatrix} = \int_7^{16} \frac{x}{\sqrt{x+9}}\, dx$$

Formula 13: $\int \frac{z}{\sqrt{az+b}}\, dz = \frac{2az - 4b}{3a^2}\sqrt{az+b} + C$

$$\boxed{\begin{array}{ll} z = x & a = 1 \\ dz = dx & b = 9 \end{array}}$$

$$\int_7^{16} \frac{x}{\sqrt{1\cdot x+9}}\, dx = \left[\frac{2(1)x - 4(9)}{3(1)^2}\sqrt{1\cdot x+9}\right]\Bigg|_7^{16}$$

$$= \left[\frac{2x-36}{3}\sqrt{x+9}\right]\Bigg|_7^{16}$$

$$= \frac{2(16)-36}{3}\sqrt{16+9} - \frac{2(7)-36}{3}\sqrt{7+9}$$

$$= \frac{32-36}{3}\sqrt{25} - \frac{14-36}{3}\sqrt{16}$$

$$= \frac{-4}{3}(5) - \frac{-22}{3}(4)$$

$$= \frac{-20}{3} + \frac{88}{3}$$

$$= \frac{68}{3}$$

$$\approx 22.667 \text{ thousand}$$

$$\approx 22,667 \text{ Newspapers}$$

From week 7 to week 16, the Times Reporter sold a total of 22,667 newspapers.

#2) Genetic Engineering

According to the problem that you are reading right now, the number of generations of bacteria needed to increase the frequency of a gene from 0.1 to 0.4 is $n = 4\int_{0.1}^{0.4} \frac{1}{x^2(1-x)}\, dx$. Find n rounded to the nearest integer.

Formula 17: $\int \frac{1}{z^2(az+b)}\, dz = -\frac{1}{b}\left(\frac{1}{z} + \frac{a}{b}\ln\left|\frac{z}{az+b}\right|\right) + C$

$$\boxed{\begin{array}{ll} z^2 = x^2 & a = -1 \\ z = x & b = 1 \\ dz = dx & \end{array}}$$

$$n = 4\int_{0.1}^{0.4} \frac{1}{x^2(-1\cdot x+1)}\, dx = 4\left[-\frac{1}{1}\left(\frac{1}{x} + \frac{-1}{1}\ln\left|\frac{x}{-1\cdot x+1}\right|\right)\right]\Bigg|_{0.1}^{0.4}$$

$$= 4\left[-\frac{1}{x} + \ln\left|\frac{x}{-x+1}\right|\right]\Bigg|_{0.1}^{0.4}$$

$$= 4\left[\left(\frac{1}{-0.4} + \ln\left|\frac{0.4}{-0.4+1}\right|\right) - \left(\frac{1}{-0.1} + \ln\left|\frac{0.1}{-0.1+1}\right|\right)\right]$$

$$= 4\left[-\frac{5}{3} + \ln\left|\frac{0.4}{0.6}\right| + 10 - \ln\left|\frac{0.1}{0.9}\right|\right]$$

$$= 4\left[7.5 + \ln\left|\frac{2}{3}\right| - \ln\left|\frac{1}{9}\right|\right]$$

$$\approx 37 \text{ generations.}$$

To increase the gene frequency from 0.1 to 0.4, it would take about 37 generations