Advanced Integration
Review Chapter 10

For #'s 1- 4 use integration by parts.

#1) $\int x^3 \ln x \, dx$

$u = \ln x \qquad dv = x^3\,dx$
$du = \frac{1}{x}dx \qquad v = \frac{1}{4}x^4$

$\int u\,dv = uv - \int v\,du$

$\int \ln x \cdot x^3 dx = \ln x \cdot \frac{1}{4}x^4 - \int \frac{1}{4}x^4 \cdot \frac{1}{x}dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$

#3) $\int xe^{8x} dx$

$u = x \qquad dv = e^{8x}dx$
$du = dx \qquad v = \frac{1}{8}e^{8x}$

$\int u\,dv = uv - \int v\,du$

$\int x \cdot e^{8x}dx = x\left(\frac{1}{8}e^{8x}\right) - \int \frac{1}{8}e^{8x}dx$

$= \frac{1}{8}xe^{8x} - \frac{1}{64}e^{8x} + C$

#2) $\int (x+7)(x-3)^4 dx$

$u = x+7 \qquad dv = (x-3)^4 dx$
$du = dx \qquad \int dv = \int p^4 dp$
$v = \frac{1}{5}p^5$
$v = \frac{1}{5}(x-3)^5$

$p = x-3$
$dp = dx$

$\int u\,dv = uv - \int v\,du$

$\int (x+7)(x-3)^4 dx = (x+7)\frac{1}{5}(x-3)^5 - \int \frac{1}{5}(x-3)^5 dx$

$= \frac{1}{5}(x+7)(x-3)^5 - \frac{1}{5}\int p^5 dp$

$= \frac{1}{5}(x+7)(x-3)^5 - \frac{1}{30}p^6 + C$

$= \frac{1}{5}(x+7)(x-3)^5 - \frac{1}{30}(x-3)^6 + C$

#4) $\int_0^1 \frac{x+1}{e^{5x}} dx$

$u = x+1 \qquad dv = e^{-5x}dx$
$du = dx \qquad v = -\frac{1}{5}e^{-5x}$

$\int u\,dv = uv - \int v\,du$

$\int (x+1)e^{-5x}dx = (x+1)\left(-\frac{1}{5}e^{-5x}\right) - \int -\frac{1}{5}e^{-5x}dx$

$= -\frac{1}{5}(x+1)e^{-5x} - \frac{1}{25}e^{-5x} + C$

$\int_0^1 \frac{x+1}{e^{5x}}dx = \left[\frac{-1}{5}(1+1)e^{-5(1)} - \frac{1}{25}e^{-5(1)}\right] - \left[\frac{-1}{5}(0+1)e^{-5(0)} - \frac{1}{25}e^{-5(0)}\right]$

$= \left[\frac{-1}{5}(1+1)e^{-5(1)} - \frac{1}{25}e^{-5(1)}\right] - \left[\frac{-1}{5}(0+1)e^{-5(0)} - \frac{1}{25}e^{-5(0)}\right]$

$= \left[\frac{-1}{5}(2)e^{-5} - \frac{1}{25}e^{-5}\right] - \left[\frac{-1}{5}(1)e^{0} - \frac{1}{25}e^{0}\right]$

$= \left[\frac{-2}{5}e^{-5} - \frac{1}{25}e^{-5}\right] - \left[-\frac{1}{5} - \frac{1}{25}\right]$

$= \left[\frac{-10}{25}e^{-5} - \frac{1}{25}e^{-5}\right] - \left[\frac{-5}{25} - \frac{1}{25}\right]$

$= \left[\frac{-10}{25}e^{-5}\right] - \left[\frac{-6}{25}\right]$

$= \frac{-11}{25}e^{-5} + \frac{6}{25}$

Use the complex integral formulas to integrate

#5) $\int \frac{1}{4-x^2}\,dx$

Formula 16: $\int \frac{1}{a^2-z^2}\,dz = \frac{1}{2a}\ln\left|\frac{a+z}{a-z}\right| + C$

$z^2 = x^2$ $\quad a^2 = 4$
$z = x$ $\quad a = 2$
$dz = dx$

$\int \frac{1}{4-x^2}\,dz = \frac{1}{2(2)}\ln\left|\frac{2+x}{2-x}\right| + C$

$\int \frac{1}{4-x^2}\,dz = \frac{1}{4}\ln\left|\frac{2+x}{2-x}\right| + C$

#6) $\int \frac{x}{x^4-9}\,dx$

Formula 15: $\int \frac{1}{z^2-a^2}\,dz = \frac{1}{2a}\ln\left|\frac{z-a}{z+a}\right| + C$

$z^2 = x^4$ $\quad a^2 = 9$
$z = x^2$ $\quad a = 3$
$dz = 2x\,dx$

$\int \frac{1}{x^4-9}\,2x\,dx = \frac{1}{2(3)}\ln\left|\frac{x^2-3}{x^2+3}\right| + C$

$2\int \frac{x}{x^4-9}\,dx = \frac{1}{6}\ln\left|\frac{x^2-3}{x^2+3}\right| + C$

$\int \frac{x}{x^4-9}\,dx = \frac{1}{12}\ln\left|\frac{x^2-3}{x^2+3}\right| + C$

#7) $\int_{-1}^{1} \frac{e^t}{e^{2t}-25}\,dt = \int_{-1}^{1} \frac{e^t}{(e^t-5)(e^t+5)}\,dt$

Formula 10: $\int \frac{1}{(az+b)(cz+d)}\,dz = \frac{1}{ad-bc}\ln\left|\frac{az+b}{cz+d}\right| + C$

$z = e^t$ $\quad a = 1$
$dz = e^t\,dt$ $\quad b = -5$
$c = 1$
$d = 5$

$\int \frac{1}{(1\cdot e^t-5)(1\cdot e^t+5)}\,e^t\,dt = \frac{1}{(1)(5)-(-5)(1)}\ln\left|\frac{1\cdot e^t+(-5)}{1\cdot e^t+5}\right|$

$= \frac{1}{5+5}\ln\left|\frac{e^t-5}{e^t+5}\right|$

$\int \frac{e^t}{e^{2t}-25}\,dt = \frac{1}{10}\ln\left|\frac{e^t-5}{e^t+5}\right|$

$\int_{-1}^{1} \frac{e^t}{e^{2t}-25}\,dt = \frac{1}{10}\ln\left|\frac{e^t-5}{e^t+5}\right|\Big|_{-1}^{1}$

$= \frac{1}{10}\left[\ln\left|\frac{e^1-5}{e^1+5}\right|\right] - \frac{1}{10}\left[\ln\left|\frac{e^{-1}-5}{e^{-1}+5}\right|\right]$

$= \frac{1}{10}\left[\ln\left|\frac{e-5}{e+5}\right|\right] - \frac{1}{10}\left[\ln\left|\frac{\frac{1}{e}-5}{\frac{1}{e}+5}\right|\right]$