Limits & Improper Integrals
11.2 – Limits Approaching $\pm\infty$

Limits as x Approaches $\pm\infty$

$x \to \infty$ x takes on arbitrary large values

$x \to -\infty$ x takes on arbitrary large values negative

What do these limits approach?

$\lim_{x\to\infty} \frac{1}{x^2} =$ 0

$x \to \infty$	$\frac{1}{x^2} \to 0$
1	$\frac{1}{1^2} = 1$
2	$\frac{1}{4} = 0.25$
4	$\frac{1}{16} = 0.625$
8	$\frac{1}{64} = 0.15625$
16	$\frac{1}{256} = 0.0039$

$\lim_{x\to\infty} \frac{1}{e^x} =$ 0

$\lim_{x\to\infty} x^2 =$ ∞, dne

$\lim_{x\to\infty} e^x =$ ∞, dne

$\lim_{x\to-\infty} x^2 =$ ∞, dne

$\lim_{x\to-\infty} e^x =$ $e^{-\infty} = \frac{1}{e^{\infty}} = 0$

Rules for x approaching infinity

$$\lim_{x\to\infty} \frac{1}{x^n} = 0, if\ n > 0$$

$$\lim_{x\to\infty} \frac{1}{e^{ax}} = 0, if\ a > 0$$

Similar Rules for x approaching *negative* infinity:

$$\lim_{x\to-\infty} \frac{1}{x^n} = 0, if\ n > 0$$

$$\lim_{x\to-\infty} e^{ax} = 0, if\ a > 0$$

For a limit to exist, it <u>must</u> be *finite*.

The following limits <u>do not exist:</u>

$$\lim_{n\to\infty} x^n = d.n.e. \qquad if\ n > 0$$

$$\lim_{x\to\infty} e^{ax} = d.n.e. \qquad if\ a > 0$$

$$\lim_{x\to\infty} \ln x = d.n.e. \qquad if\ n > 0$$

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Ex. A: Evaluating Limits

#1) $\lim_{b\to\infty}\frac{1}{b} = \frac{1}{\infty} = 0$

#2) $\lim_{b\to\infty}(5+\frac{1}{b}) = 5+\frac{1}{\infty}$

$= 5+0$

$= 5$

#3) $\lim_{b\to\infty}(e^{-3b}-2) = e^{-3(\infty)}-2$

$= \frac{1}{e^{\infty}}-2$

$= \frac{1}{\infty}-2$

$= 0-2$

$= -2$

#4) $\lim_{b\to-\infty}(5+7^{b}) = 5+7^{-\infty}$

$= 5+\frac{1}{7^{\infty}}$

$= 5+\frac{1}{\infty}$

$= 5+0$

$= 5$

#5) $\lim_{b\to-\infty}(e^{3b}-2) = e^{3(-\infty)}-2$

$= \frac{1}{e^{\infty}}-2$

$= 0-2$

$= -2$

Ex. B: Finding Whether a Limit Exists

#1) $\lim_{b\to\infty} b^2 = \infty^2$

$= \infty$, dne.

#2) $\lim_{b\to\infty}(\sqrt{b}+5) = \sqrt{\infty}+5$

$= \infty+5$

$= \infty$, dne

#3) $\lim_{b\to-\infty}(5-3^{-b}) = 5-3^{-(-\infty)}$

$= 5-3^{\infty}$

$= 5-\infty$

$= \infty$, dne

#4) $\lim_{b\to\infty}(\sqrt[3]{b}+1) = \sqrt[3]{\infty}+1$

$= \infty+1$

$= \infty$, dne

#5) $\lim_{b\to-\infty}(-6+2^{b}) = -6+2^{-\infty}$

$= 6+\frac{1}{2^{\infty}}$

$= 6+\frac{1}{\infty}$

$= 6+0$

$= 6$