

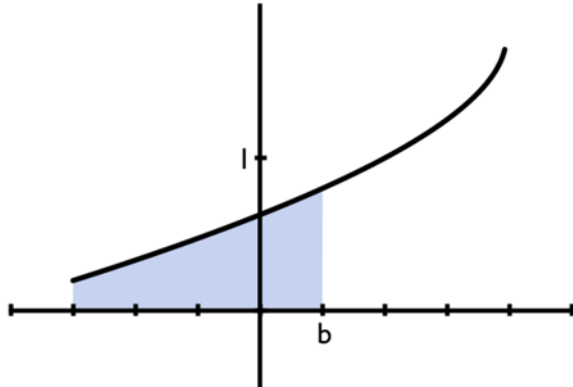
Limits & Improper Integrals

11.4 – Improper Integrals: Integrating to $-\infty$

Improper Integrals – Integrating to $-\infty$

For any continuous and nonnegative function f when $x \leq a$, we define

$$\int_{-\infty}^b f(x) dx = \lim_{a \rightarrow -\infty} \int_a^b f(x) dx$$



provided that the limit exists.

Convergent or Divergent?

If the limit exists, it is *convergent*.

If the limit does not exist, it is *divergent*.

Improper Integral

Improper integral = Integrating with infinity or negative infinite.

$$\int_{-\infty}^1 \frac{1}{x^2} dx$$

Procedure:

#1. Find the area under the curve from a to b .

#2. Evaluate the $\lim_{a \rightarrow -\infty} \int_a^b f(x) dx$ to find the area arbitrarily far from the left.

$$\int_{-\infty}^b f(x) dx = \lim_{a \rightarrow -\infty} \int_a^b f(x) dx$$

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Ex A: Integrating to $-\infty$

#1) Evaluate $\int_{-\infty}^3 8e^{2x} dx$

$$\begin{aligned} &= \lim_{a \rightarrow -\infty} \int_a^3 8e^{2x} dx \\ &= \lim_{a \rightarrow -\infty} 4e^{2x} \Big|_a^3 \\ &= \lim_{a \rightarrow -\infty} [4e^{2(3)} - 4e^{2(a)}] \\ &= \lim_{a \rightarrow -\infty} [4e^6 - 4e^{2a}] \\ &= 4e^6 - 4e^{2(-\infty)} \\ &= 4e^6 - 4e^{\infty} \\ &= 4e^6 - \frac{4}{e^{\infty}} \\ &= 4e^6 - 0 \\ &= 4e^6 \end{aligned}$$

#2) Evaluate $\int_{-\infty}^1 12e^{4x} dx$

$$\begin{aligned} &= \lim_{a \rightarrow -\infty} \int_a^1 12e^{4x} dx \\ &= \lim_{a \rightarrow -\infty} [12e^{4(1)} - 12e^{4(a)}] \\ &= \lim_{a \rightarrow -\infty} [12e^4 - 12e^{4a}] \\ &= 12e^4 - 12e^{4(-\infty)} \\ &= 12e^4 - \frac{12}{e^{\infty}} \\ &= 12e^4 - 0 \\ &= 12e^4 \end{aligned}$$

$$\lim_{b \rightarrow -\infty} e^{ax} = 0$$