Limits and Improper Integrals
Chapter 11 Review

#1) $\lim_{x\to 0} \frac{8x}{\sin(x)} = 8 \lim_{x\to 0} \frac{x}{\sin(x)}$

$= 8 \cdot 1$

$= 8$

#2) $\lim_{x\to \frac{5}{2}} \frac{\sin(2x-5)}{10x-25}$

$= \lim_{x\to 5/2} \frac{\sin(2x-5)}{5(2x-5)}$

$= \frac{1}{5} \lim_{x\to 5/2} \frac{\sin(2x-5)}{2x-5}$

$= \frac{1}{5} \lim_{\theta\to 0} \frac{\sin\theta}{\theta}$

$= \frac{1}{5} \cdot 1$

$= \frac{1}{5}$

if $x \to \frac{5}{2}$, then $(2x-5) \to 0$ so, let's use a substitution $\theta = 2x-5$

#3) $\lim_{x\to 0} \frac{\cos(x)-1}{6x} = \frac{1}{6} \lim_{x\to 0} \frac{\cos(x)-1}{x}$

$= \frac{1}{6} \cdot 0$

$= 0$

#4) $\lim_{x\to 0} \frac{\cos(27x)-1}{9x} = 3 \lim_{x\to 0} \frac{\cos(27x)-1}{27x}$

$= 3 \cdot 0$

$= 0$

#5) $\lim_{x\to \frac{\pi}{2}} \frac{\cos(x)}{x-\frac{\pi}{2}} = \frac{\cos(\frac{\pi}{2})}{\frac{\pi}{2}-\frac{\pi}{2}} = \frac{0}{0}$

L'Hopital

$\lim_{x\to \frac{\pi}{2}} \frac{[\cos(x)]'}{(x-\frac{\pi}{2})'}$

$= \lim_{x\to \frac{\pi}{2}} \frac{-\sin(x)}{1}$

$= -\sin(\frac{\pi}{2})$

$= -1$

#6) $\lim_{x\to \pi} \frac{\sin(x)}{x-\pi} = \frac{\sin\pi}{\pi-\pi} = \frac{0}{0}$

L'Hopital

$\lim_{x\to \pi} \frac{[\sin(x)]'}{[x-\pi]'} = \lim_{x\to \pi} \frac{\cos(x)}{1}$

$= \lim_{x\to \pi} \cos\pi$

$= \cos\pi$

$= -1$

#7) $\lim_{x\to \infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$

L'Hopital

$\lim_{x\to \infty} \frac{(\ln x)'}{x'}$

$= \lim_{x\to \infty} \frac{\frac{1}{x}}{1}$

$= \lim_{x\to \infty} \frac{1}{x}$

$= $ ☺

Given an improper integral, evaluate it and state whether it is convergent or divergent.

#8) $\int_2^\infty \frac{1}{x^2}dx = \lim_{b\to\infty} \int_2^b x^{-2}dx$

$= \lim_{b\to\infty} \left(-x^{-1}\right)\Big|_2^b$

$= \lim_{b\to\infty} \left(\frac{-1}{x}\right)\Big|_2^b$

$= \lim_{b\to\infty} \left[\frac{-1}{b} - \frac{-1}{2}\right]$

$= \frac{-1}{\infty} + \frac{1}{2}$

$= 0 + \frac{1}{2}$

$= \frac{1}{2}$, convergent

#9) $\int_0^\infty \frac{2x}{x^2+1}dx = \lim_{b\to\infty} \int_0^b \frac{2x}{x^2+1}dx$

$\boxed{\begin{array}{l} u = x^2+1 \\ du = 2x\,dx \\ \frac{du}{2x} = dx \end{array}}$

$= \lim_{b\to\infty} \int_{x=0}^{x=b} \frac{2x}{u}\frac{du}{2x}$

$= \lim_{b\to\infty} \int_{x=0}^{x=b} \frac{1}{u}du$

$= \lim_{b\to\infty} \ln|u|\Big|_{x=0}^{x=b}$

$= \lim_{x\to\infty} \ln|x^2+1|\Big|_0^b$

$= \lim_{x\to\infty} \left[\ln|(b)^2+1| - \ln|(0)^2+1|\right]$

$= \left[\ln|\infty^2+1| - \ln|0+1|\right]$

$= \ln|\infty| - \ln|1|$

$= \infty - (0)$

$= \infty - 0$

$= \infty$, dne divergent

Given an improper integral, evaluate it and state whether it is convergent or divergent.

#10) $\int_{-\infty}^{0} e^{2x}\,dx = \lim_{a\to-\infty} \int_a^0 e^{2x}\,dx$

$$= \lim_{a\to-\infty} \left(\frac{1}{2}e^{2x}\right)\Big|_a^0$$

$$= \lim_{a\to-\infty} \left[\frac{1}{2}e^{2(0)} - \frac{1}{2}e^{2a}\right]$$

$$= \lim_{a\to-\infty} \left[\frac{1}{2}e^{0} - \frac{1}{2}e^{2a}\right]$$

$$= \frac{1}{2}(1) - \frac{1}{2}e^{2(-\infty)}$$

$$= \frac{1}{2} - \frac{1}{2e^{\infty}}$$

$$= \frac{1}{2} - 0$$

$$= \frac{1}{2}, \text{ convergent}$$

#11) $\int_{-\infty}^{0} \frac{1}{10-x}\,dx = \lim_{a\to-\infty} \int_a^0 (10-x)^{-1}\,dx$

$$\boxed{\begin{array}{l} u = 10-x \\ du = -dx \\ -du = dx \end{array}}$$

$$= \lim_{a\to-\infty} \int_{x=a}^{x=0} (u)^{-1}(-du)$$

$$= -\lim_{a\to-\infty} \int_{x=a}^{x=0} u^{-1}\,du$$

$$= -\lim_{a\to-\infty} \ln|u|\Big|_{x=a}^{x=0}$$

$$= -\lim_{a\to-\infty} \ln|10-x|\Big|_a^0$$

$$= -\lim_{a\to-\infty} \left[\ln|10-(0)| - \ln|10-a|\right]$$

$$= -\lim_{a\to-\infty} \left[\ln|10| - \ln|10-a|\right]$$

$$= -\left[\ln 10 - \ln|10-(-\infty)|\right]$$

$$= -\ln 10 + \ln|\infty|$$

$$= -\ln 10 + \infty$$

$$= \infty, \text{ dne divergent}$$

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Given an improper integral, evaluate it and state whether it is convergent or divergent.

#12) $\int_{-\infty}^{\infty} \frac{e^x}{12+e^x}\,dx$

$$= \int_{-\infty}^{\infty} \frac{e^x}{u} \frac{du}{e^x}$$

$$= \int_{-\infty}^{\infty} \frac{1}{u}\,du$$

$$\boxed{\begin{array}{l} u = 12+e^x \\ du = e^x\,dx \\ \frac{du}{e^x} = dx \end{array}}$$

$$= \lim_{a\to-\infty} \int_{x=a}^{x=0} \frac{1}{u}\,du + \lim_{b\to\infty} \int_{x=0}^{x=b} \frac{1}{u}\,du$$

$$= \lim_{a\to-\infty} \ln|u|\Big|_{x=a}^{x=0} + \lim_{b\to\infty} \ln|u|\Big|_{x=0}^{x=b}$$

$$= \lim_{a\to-\infty} \ln|12+e^x|\Big|_{a}^{0} + \lim_{b\to\infty} \ln|12+e^x|\Big|_{0}^{b}$$

$$= \left[\ln|12+e^0| - \ln|12+e^{-\infty}|\right] + \left[\ln|12+e^{\infty}| - \ln|12+e^0|\right]$$

$$= \left[\ln|12+1| - \ln\left|12+\frac{1}{e^{\infty}}\right|\right] + \left[\ln|\infty| - \ln|12+1|\right]$$

$$= \left[\ln|13| - \ln|12+0|\right] + \left[\infty - \ln|13|\right]$$

$$= \left[\ln 13 - \ln 12\right] + \left[\infty - \ln|13|\right]$$

$$= -\ln 12 + \infty$$

$= \infty$, dne divergent