Limits & Continuity
Chapter 1 Review

Find the following limits *without* using a graphing calculator or making tables.

#1) $\lim_{x\to -1} \frac{x^2}{2x} = \frac{(-1)^2}{2(-1)} = \frac{1}{-2}$

#2) $\lim_{h\to 0} \frac{x^4h - xh^2}{h} = \lim_{h\to 0} \frac{h(x^4 - xh)}{h}$

$= \lim_{h\to 0} (x^4 - xh)$

$= x^4 - x(0)$

$\lim_{h\to 0} \frac{x^4h - xh^2}{h} = x^4$

Answer each question concerning piecewise functions.

#3) $f(x) = \begin{cases} -x+4, & if\ x < 4 \\ x-5, & if\ x \geq 4 \end{cases}$

a. $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (-x+4)$

$= -(4)+4$

$= 0$

b. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} (x-5)$

$= (4) - 5$

$= -1$

c. $\lim_{x\to 4} f(x) =$ d.n.e.

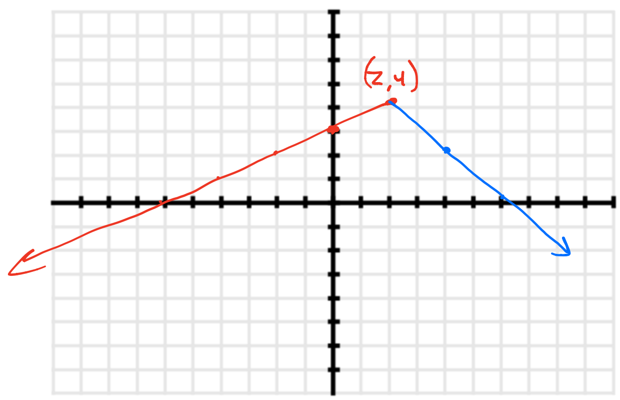
#4) For the following piecewise function:

$$f(x) = \begin{cases} \frac{1}{2}x + 3, & if\ x \leq 2 \\ -x + 6, & if\ x > 2 \end{cases}$$

a. Draw its graph

x	$\frac{1}{2}x+3$
2	4
0	3

x	$-x+6$
2	4
4	2



b. Find the limits as x approaches 2 from the left.

$\lim_{x\to 2^-} f(x) = 4$

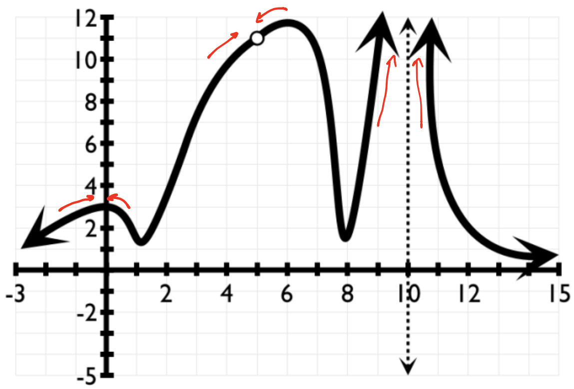
c. Find the limits as x approaches 2 from the right.

$\lim_{x\to 2^+} f(x) = 4$

d. Is it continuous at x = 2? If not, why?

yes :)

#5) Find each limit. Assume that each limit that does exist is an integer. (There is no work to be shown)



a. $\lim_{x\to 0^-} f(x) = 3$

b. $\lim_{x\to 0^+} f(x) = 3$

c. $\lim_{x\to 0} f(x) = 3$

d. $\lim_{x\to 5^-} f(x) = 11$

e. $\lim_{x\to 5^+} f(x) = 11$

f. $\lim_{x\to 5} f(x) = 11$

g. $\lim_{x\to 10^-} f(x) =$ ∞, dne

h. $\lim_{x\to 10^+} f(x) =$ ∞, dne

i. $\lim_{x\to 10} f(x) =$ ∞, dne

#6) Find the equation for the tangent line to the curve $f(x) = \frac{1}{2}x^2$ at $x = 1$. Write your equation in slope-intercept form.

1 Point @ x = 1

$$f(1) = \frac{1}{2}(1)^2 = \frac{1}{2}(1)$$
$$f(1) = \frac{1}{2}$$
$$\left(1, \frac{1}{2}\right)$$

Slope

2
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\left[\frac{1}{2}(x+h)^2\right] - \left[\frac{1}{2}x^2\right]}{h}$$
$$= \lim_{h\to 0} \frac{\frac{1}{2}(x^2+2hx+h^2) - \frac{1}{2}x^2}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{\frac{1}{2}x^2} + hx + \frac{1}{2}h^2 - \cancel{\frac{1}{2}x^2}}{h}$$
$$= \lim_{h\to 0} \frac{hx + \frac{1}{2}h^2}{h}$$
$$= \lim_{h\to 0} \frac{h(x + \frac{1}{2}h)}{h}$$
$$= \lim_{h\to 0} \left(x + \frac{1}{2}h\right)$$
$$= x + \frac{1}{2}(0)$$
$$f'(x) = x$$

3 Slope @ x = 1

$$f'(1) = 1$$

4 Point-Slope form

$$y - y_1 = m(x - x_1)$$
$$y - \left(\frac{1}{2}\right) = 1\left(x - (1)\right)$$
$$y - \frac{1}{2} = x - 1$$
$$y = x - \frac{1}{2}$$

#7) Find the equation for the tangent line to the curve $f(x) = x^2 - 8x + 5$ at $x = 2$. Write your equation in slope-intercept form.

① Point @ $x = 2$

$$\boxed{\begin{aligned} f(2) &= (2)^2 - 8(2) + 5 \\ &= 4 - 16 + 5 \\ &= -12 + 5 \\ f(2) &= -7 \\ &(2, -7) \end{aligned}}$$

② Slope

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^2 - 8(x+h) + 5] - [x^2 - 8x + 5]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2hx + h^2 \cancel{- 8x} - 8h \cancel{+ 5} \cancel{- x^2} \cancel{+ 8x} \cancel{- 5}}{h}$$

$$= \lim_{h \to 0} \frac{2hx + h^2 - 8h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h - 8)}{h}$$

$$= \lim_{h \to 0} (2x + h - 8)$$

$$= 2x + (0) - 8$$

$$\frac{dy}{dx} = 2x - 8$$

③ Slope @ $x = 2$

$$\frac{dy(2)}{dx} = 2(2) - 8$$

$$= 4 - 8$$

$$\frac{dy}{dx} = -4$$

④ Point-Slope form

$$y - y_1 = m(x - x_1)$$

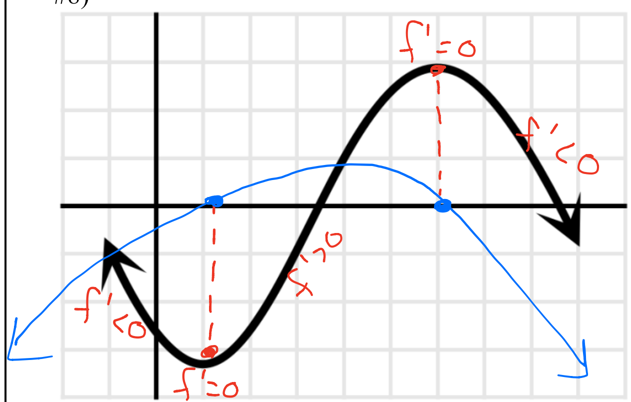
$$y - (-7) = -4(x - (2))$$

$$y + 7 = -4x + 8$$

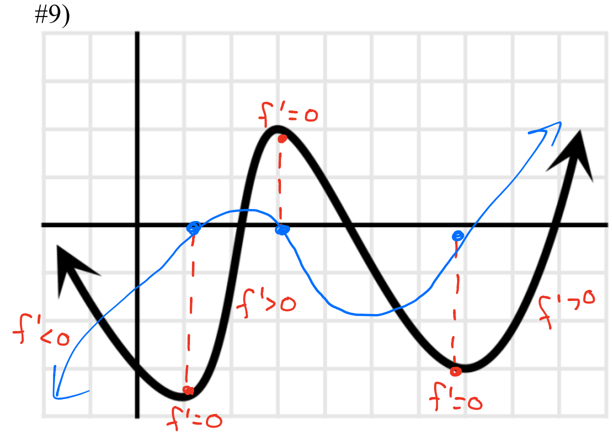
$$y = -4x + 1$$

Given the graph of a function, sketch in the graph of its derivative function.

#8)



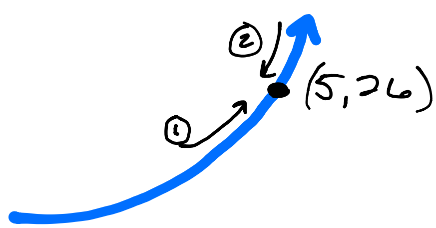
#9)



#10) Often times, problems will ask for the derivative without using the word "derivative". We have learned two interpretations of a derivative. What are these two interpretations?

Derivative means the slope of a tangent line.
Derivative means the instantaneous rate of change.

#11) $\lim_{x \to 5}(x^2 + 1) = 26$ is read "the limit of $x^2 + 1$, as x approaches 5, is 26." Use sentences and graphs to illustrate the meaning of said statement.

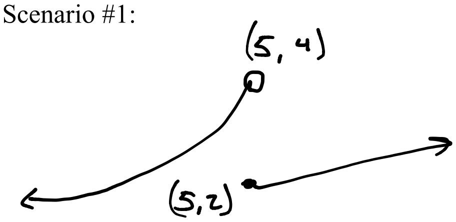


1 As x gets closer and closer to 5 from the left, Y gets closer and closer to 26.

2 As x approaches 5 from the right Y approaches 26.

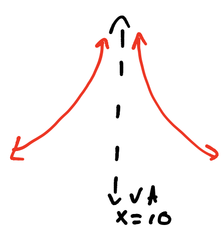
#12) Give 2 specific scenarios of when a limit would not exist and explain why. You *may* use graphs to illustrate your point.

Scenario #1:



The limit would not exist at x=5.
As $x \to 5^-$ the limit is 4.
As $x \to 5^+$ the limit is 2.
The two-sided limit does not exist because the left and right limits do not agree

Scenario #2:



The limit does not exist @ x=10.
As $x \to 10$ from the left or the right, the y-value approaches infinity.
For a limit to exist, it must approach a single number.
(∞ is not a number)