Basic Derivative Rules
2.1A – Power Rule

A: The Power Rule

#1) Find $\frac{d}{dx}x^5$ $= 5x^4$

#2) Find $\frac{dx^{700}}{dx}$

$= 700x^{699}$

#3) If $y = x^{-2/3}$, find y'

$y' = -\frac{2}{3}x^{-5/3}$

#4) If $y = 7\sqrt{x}$, find y'

$y = 7x^{\frac{1}{2}}$ $y' = \frac{7}{2}x^{-\frac{1}{2}}$

$y' = \frac{7}{2\sqrt{x}}$

#5) Find $\frac{d}{dx}\left(\frac{x^2}{5}\right) = \frac{d}{dx}\left(\frac{1}{5}x^2\right)$

$= \frac{2}{5}x$

#6) Find $\frac{d}{dx}\left(\frac{6}{x^6}\right)$ $= \frac{d}{dx}(6x^{-6})$

$= -36x^{-7}$

$\frac{d}{dx}\left(\frac{6}{x^6}\right) = \frac{-36}{x^7}$

#7) If $y = \sqrt[6]{x}$, find y'

$y = x^{1/6}$

$y' = \frac{1}{6}x^{-5/6}$

$y' = \frac{1}{6\sqrt[6]{x^5}}$

#8) If $y = 9x$, find y'

$y' = 9$

#9) Find $\frac{d}{dx}(5x^3 - 5x^2 + 7x + 9)$ $= 15x^2 - 10x + 7$

#10) If $y = 5x^{-7} + 9x^{-3} - 9$, find y'

$y' = -35x^{-8} - 27x^{-4}$

#11) If $f(x) = \pi x^4 - x^\pi$, find $f'(x)$

$f'(x) = 4\pi x^3 - \pi x^{\pi-1}$

#12) Find $\frac{d}{dx}\left(\frac{8}{\sqrt[4]{x}} - \frac{5}{\sqrt{x}}\right) = \frac{d}{dx}\left(8x^{-\frac{1}{4}} - 5x^{-\frac{1}{2}}\right)$

$= -2x^{-5/4} + \frac{5}{2}x^{-3/2}$

$\frac{d}{dx}\left(\frac{8}{\sqrt[4]{x}} - \frac{5}{\sqrt{x}}\right) = \frac{-2}{\sqrt[4]{x^5}} + \frac{5}{2\sqrt{x^3}}$

#13) Find $\frac{d}{dx}\left(\frac{x^3+x^5}{x}\right) = \frac{d}{dx}\left[\frac{x(x^2+x^4)}{x}\right]$

$= \frac{d}{dx}(x^2+x^4)$

$= 2x+4x^3$

#14) Find *f ’(x)* if $f(x) = \sqrt[6]{x} + \frac{3}{\sqrt[7]{x^5}}$

$f(x) = x^{\frac{1}{6}} + 3x^{-5/7}$

$f'(x) = \frac{1}{6}x^{-5/6} - \frac{15}{7}x^{-12/7}$

$f'(x) = \frac{1}{6\sqrt[6]{x^5}} - \frac{15}{7\sqrt[7]{x^{12}}}$

#15) Find $\frac{d}{dx}90{,}000{,}000{,}000 = 0$

#16) If $f(x) = 0$, find *f ’(x)*

$f' = 0$

B: Find each equation in slope-intercept form.

#17) Find the equation of the tangent line to $f(x) = x^2 - 4x + 8$ at x = 4

Point @ x=4

$f(x) = x^2 - 4x + 8$

$f(4) = (4)^2 - 4(4) + 8$

$f(4) = 16 - 16 + 8$

$f(4) = 8$

$(4, 8)$

Slope @ x=4

$f'(x) = 2x - 4$

$f'(4) = 2(4) - 4$

$= 8 - 4$

$f'(4) = 4$

$m = 4$

Point - Slope Form

$y - y_1 = m(x - x_1)$

$y - (8) = 4(x - (4))$

$y - 8 = 4x - 16$

$y = 4x - 8$

#18) Find the equation of the tangent line to $f(x) = 3x^2 + 8$ at x = -2

Point @ x=-2

$f(x) = 3x^2 + 8$

$f(-2) = 3(-2)^2 + 8$

$= 3(4) + 8$

$= 12 + 8$

$f(-2) = 20$

$(-2, 20)$

Slope @ x=-2

$f'(x) = 6x$

$f'(-2) = 6(-2)$

$f'(-2) = -12$

$m = -12$

Point - Slope form

$y - y_1 = m(x - x_1)$

$y - (20) = -12(x - (-2))$

$y - 20 = -12x - 24$

$y = -12x - 4$

Ex C: Evaluate each derivative

#19) If $f(x) = x^3$, find $f'(-2)$.

$f'(x) = 3x^2$ $\quad f'(-2) = 3(-2)^2$

$= 3(4)$

$f'(-2) = 12$

#20) If $f(r) = \frac{4}{3}\pi r^3$, find $f'(2)$.

$f'(r) = 4\pi r^2$ $\quad f'(2) = 4\pi(2)^2$

$= 4\pi(4)$

$f'(2) = 16\pi$

#21) If $f(x) = \sqrt[3]{x} + \frac{1}{\sqrt{x}}$, find $f'(1)$.

$f(x) = x^{\frac{1}{3}} + x^{-\frac{1}{2}}$

$f'(x) = \frac{1}{3}x^{-2/3} - \frac{1}{2}x^{-3/2}$

$f'(x) = \frac{1}{3\sqrt[3]{x^2}} - \frac{1}{2\sqrt{x^3}}$

$f'(1) = \frac{1}{3\sqrt[3]{(1)^2}} - \frac{1}{2\sqrt{(1)^3}}$

$= \frac{1}{3(1)} - \frac{1}{2(1)}$

$= \frac{1}{3} - \frac{1}{2}$

$= \frac{2}{6} - \frac{3}{6}$

$f'(1) = -\frac{1}{6}$

#22) If $f(x) = 6\sqrt{x}$, find $f'(8)$.

$f(x) = 6x^{\frac{1}{2}}$

$f'(x) = 3x^{-\frac{1}{2}}$

$f'(x) = \frac{3}{\sqrt{x}}$

$f'(8) = \frac{3}{\sqrt{8}}$

$f'(8) = \frac{3}{2\sqrt{2}}$

#23) If $f(x) = \frac{4}{x^2}$, find $f'(3)$.

$f(x) = 4x^{-2}$

$f'(x) = -8x^{-3}$

$f'(x) = \frac{-8}{x^3}$

$f'(3) = \frac{-8}{(3)^3}$

$f'(3) = \frac{-8}{27}$

#24) If $f(x) = x^3$, find $\left.\frac{df}{dx}\right|_{x=-3}$

$\left.\frac{d}{dx}(x^3)\right|_{x=-3} = \left.3x^2\right|_{x=-3}$

$= \left.3x^2\right|_{x=-3}$

$= 3(-3)^2$

$= 3(9)$

$\left.\frac{d}{dx}(x^3)\right|_{x=-3} = 27$

#25) If $f(r) = \frac{4}{3}\pi r^3$, find $\left.\frac{df}{dr}\right|_{r=4}$

$\left.\frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\right|_{x=4} = \left.4\pi r^2\right|_{x=4}$

$= 4\pi(4)^2$

$= 4\pi(16)$

$\left.\frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\right|_{x=4} = 216\pi$

#26) If $f(x) = \sqrt[3]{x} + \frac{1}{\sqrt{x}}$ find $\left.\frac{df}{dx}\right|_{x=2}$

$\left.\frac{d}{dx}\left(x^{\frac{1}{3}} + x^{-\frac{1}{2}}\right)\right|_{x=2} = \left.\left(\frac{1}{3}x^{-2/3} - \frac{1}{2}x^{-3/2}\right)\right|_{x=2} = \left.\left(\frac{1}{3\sqrt[3]{x^2}} - \frac{1}{2\sqrt{x^3}}\right)\right|_{x=2}$

$= \frac{1}{3\sqrt[3]{(2)^2}} - \frac{1}{2\sqrt{(2)^3}}$

$= \frac{1}{3\sqrt[3]{4}} - \frac{1}{2\sqrt{8}}$

$\left.\frac{d}{dx}\left(x^{\frac{1}{3}} + x^{-\frac{1}{2}}\right)\right|_{x=2} = \frac{1}{3\sqrt[3]{4}} - \frac{1}{4\sqrt{2}}$

Basic Derivative Rules

2.1A – Power Rule

#27) If $f(x) = 6\sqrt{x^5}$, find $\left.\frac{df}{dx}\right|_{x=16}$

$$\left.\frac{d}{dx}\left(6x^{5/2}\right)\right|_{x=16} = \left.\left(\cancel{6}^{3}\cdot\frac{5}{\cancel{2}}x^{3/2}\right)\right|_{x=16} = \left.\left(15\left(\sqrt{x}\right)^3\right)\right|_{x=16}$$

$$= 15\left(\sqrt{16}\right)^3$$

$$= 15(4)^3$$

$$= 15(64)$$

$$\left.\frac{d}{dx}\left(6\sqrt{x^5}\right)\right|_{x=16} = 960$$

#28) If $f(x) = \frac{1}{x^{-5}}$, find $\left.\frac{df}{dx}\right|_{x=2}$

$$\left.\frac{d}{dx}\left(x^5\right)\right|_{x=2} = \left.5x^4\right|_{x=2}$$

$$= 5(2)^4$$

$$= 5(16)$$

$$\left.\frac{d}{dx}\left(x^5\right)\right|_{x=2} = 80$$

D: Find each equation in slope-intercept form

#29) Find the equation of the tangent line to $f(x) = 4x^2 - 10x + 81$ at x = 1

Point @ x=1

$f(x) = 4x^2 - 10x + 81$
$f(1) = 4(1)^2 - 10(1) + 81$
$= 4 - 10 + 81$
$= 4 + 71$
$f(1) = 75$
$(1, 75)$

Slope @ x=1

$f'(x) = 8x - 10$
$f'(1) = 8(1) - 10$
$= 8 - 10$
$f'(1) = -2$
$m = -2$

Point-Slope Form

$y - y_1 = m(x - x_1)$
$y - (75) = -2(x - (1))$
$y - 75 = -2x + 2$
$y = -2x + 77$

#30) Find the equation of the tangent line to $f(x) = -2x^2 + 3x + 1$ at x = -5

Point @ x=-5

$f(x) = -2x^2 + 3x + 1$
$f(-5) = -2(-5)^2 + 3(-5) + 1$
$= -2(25) - 15 + 1$
$= -50 - 14$
$f(-5) = -64$
$(-5, -64)$

Slope @ x=-5

$f'(x) = -4x + 3$
$f'(-5) = -4(-5) + 3$
$= 20 + 3$
$f'(-5) = 23$
$m = 23$

Point-Slope Form

$y - y_1 = m(x - x_1)$
$y - (-64) = 23(x - (-5))$
$y + 64 = 23x + 115$
$y = 23x + 51$