

Basic Derivative Rules

2.1 – Power Rule

Leibniz's Notation

Power Rule

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

Derivative of a Constant

$$\frac{d}{dx} c = 0$$

A Special Application of the Power Rule

$$\frac{d}{dx} x = 1$$

Constant Multiple Rule

$$\frac{d}{dx} [c \cdot f(x)] = c \cdot \frac{d}{dx} [f(x)]$$

Sum-Difference Rule

$$\frac{d}{dx} [f(x) \pm g(x)] = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$$

Evaluating

$$\left. \frac{df}{dx} \right|_{x=2}$$

Evaluate $g(t)$ at 4 $\left. \frac{dg}{dt} \right|_{t=4}$

Evaluate w^3 at 1 $\left. \frac{dw}{dx} \right|_{x=1}$

Evaluate $f(x)$ at 0 $\left. \frac{df}{dx} \right|_{x=0}$

Newton's Notation

Power Rule

$$\text{If } f(x) = x^n, \text{ then } f'(x) = n \cdot x^{n-1}$$

Derivative of a Constant

$$\text{If } f(x) = c, \text{ then } f'(x) = 0.$$

A Special Application of the Power Rule

$$\text{If } f(x) = x, \text{ then } f'(x) = 1$$

Constant Multiple Rule

$$[c \cdot f(x)]' = c \cdot f'(x)$$

Sum-Difference Rule

$$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$$

Evaluating

$$f'(2)$$

Evaluate $g(t)$ at 4 $g'(4)$

Evaluate w^3 at 1 $\xi'(1)$

Evaluate $f(x)$ at 0 $\xi'(0)$

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A: The Power Rule ⁹

$$\#1) \frac{d}{dx} x^{10} = 10x^9$$

$$\#2) \frac{dx^{50}}{dx} = 50x^{49}$$

#3) If $y = x^{-8/3}$, find y'

$$y' = -\frac{8}{3}x^{-11/3}$$

#4) If $y = 8\sqrt{x}$, find y'

$$y = 8x^{1/2}$$
$$y' = 4x^{-1/2}$$
$$y' = \frac{4}{\sqrt{x}}$$

$$\#5) \frac{d}{dx} \left(\frac{x^2}{6} \right) = \frac{d}{dx} \left(\frac{1}{6}x^2 \right)$$
$$= \frac{1}{3}x$$

$$\#6) \frac{d}{dx} 90,000,000,000 = 0$$

$$\#7) \frac{d}{dx} \left(\frac{6}{x^9} \right) = \frac{d}{dx} (6x^{-9})$$
$$= -54x^{-10}$$
$$= \frac{-54}{x^{10}}$$

#8) If $y = \sqrt[3]{x}$, find y'

$$y = x^{1/3}$$
$$y' = \frac{1}{3}x^{-2/3}$$
$$y' = \frac{1}{3\sqrt[3]{x^2}}$$

#9) If $y = 6x$, find y'

$$y' = 6$$

$$\#10) \frac{d}{dx} \left(x^3 - \frac{5}{2}x^2 + 4x - 6 \right)$$
$$= 3x^2 - 5x + 4$$

#11) If $y = 5x^{-7} + 9x^{-3} - 9$, find y'

$$y' = -35x^{-8} - 27x^{-4}$$

#12) If $f(x) = 0$, find $f'(x)$

$$f'(x) = 0$$

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Ex B: Evaluating a Derivative

#1) If $h(x) = 6\sqrt{x}$, find $h'(8)$.

$$\begin{aligned} h(x) &= 6x^{\frac{1}{2}} \\ h'(x) &= 3x^{-\frac{1}{2}} \\ h'(x) &= \frac{3}{\sqrt{x}} \\ h'(8) &= \frac{3}{\sqrt{8}} \\ h'(8) &= \frac{3}{2\sqrt{2}} \end{aligned}$$

#2) If $f(r) = \frac{4}{3}\pi r^3$, find $\frac{df}{dr}\bigg|_{r=4}$

$$\begin{aligned} \frac{df}{dr}\bigg|_{r=4} &= 4\pi r^2\bigg|_{r=4} \\ &= 4\pi(4)^2 \\ &= 4\pi(16) \\ \frac{df}{dr}\bigg|_{r=4} &= 64\pi \end{aligned}$$

#3) If $f(x) = ex^6 - x^e$, find $f'(5)$

$$\begin{aligned} f'(x) &= 6ex^5 - ex^{e-1} \\ f'(5) &= 6e(5)^5 - e(5)^{e-1} \\ f'(5) &= 6(3125)e - 5^{e-1}e \\ f'(5) &= 18,750e - 5^{e-1}e \end{aligned}$$

$$\begin{aligned} \#4) \frac{d}{dw} \left(\frac{8}{\sqrt[4]{w}} - \frac{5}{\sqrt{w}} \right) \bigg|_{w=81} &= \frac{d}{dw} (8w^{-\frac{1}{4}} - 5w^{-\frac{1}{2}}) \bigg|_{w=81} \\ &= (2w^{-\frac{5}{4}} + \frac{5}{2}w^{-\frac{3}{2}}) \bigg|_{w=81} \\ &= \left(\frac{2}{4\sqrt[4]{w^5}} + \frac{5}{2\sqrt{w^3}} \right) \bigg|_{w=81} \\ &= \frac{2}{4(81)^{\frac{5}{4}}} + \frac{5}{2(\sqrt{81})^3} \\ &= \frac{2}{27 \cdot 81} + \frac{5}{2(729)} \\ &= \frac{2 \cdot 5^4}{27 \cdot 5^4} + \frac{5}{1458} \\ &= \frac{108}{1458} + \frac{5}{1458} \\ &= \frac{113}{1458} \end{aligned}$$

#5) $\frac{d}{dt} \left(\frac{t^4+t^5}{t+1} \right) \bigg|_{t=10}$

$$\begin{aligned} &= \frac{d}{dt} \left[\frac{t^4(t+1)}{t+1} \right] \bigg|_{t=10} \\ &= \frac{d}{dt} (t^4) \bigg|_{t=10} \\ &= 4t^3 \bigg|_{t=10} \\ &= 4(10)^3 \\ &= 4 \cdot (1000) \\ &= 4000 \end{aligned}$$

#6) Find $v'(1)$ if $v(t) = \sqrt[6]{t^7} - \frac{5}{\sqrt[7]{t^5}}$

$$\begin{aligned} v(t) &= t^{\frac{7}{6}} - 5t^{-\frac{5}{7}} \\ v'(t) &= \frac{7}{6}t^{-\frac{1}{6}} + \frac{25}{7}t^{-\frac{12}{7}} \\ v'(t) &= \frac{7}{6\sqrt[6]{t}} + \frac{25}{7\sqrt[7]{t^{12}}} \\ v'(1) &= \frac{7}{6\sqrt[6]{1}} + \frac{25}{7\sqrt[7]{1^{12}}} \\ &= \frac{7}{6(1)} + \frac{25}{7(1)} \\ &= \frac{7 \cdot 7}{6 \cdot 7} + \frac{25 \cdot 6}{7 \cdot 6} \\ &= \frac{49}{42} + \frac{150}{42} \\ v'(1) &= \frac{199}{42} \end{aligned}$$

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C: Find each equation in slope-intercept form.

#1) Find the equation of the tangent line to the function $f(x) = 3x^2 - x + 8$ at $x = -5$

Point @ $x = -5$	Slope @ $x = -5$
$f(-5) = 3(-5)^2 - (-5) + 8$	$f'(x) = 6x - 1$
$= 3(25) + 5 + 8$	$f'(-5) = 6(-5) - 1$
$= 75 + 13$	$= -30 - 1$
$f(-5) = 88$	$f'(-5) = -31$

Point-slope form @ $(-5, 88)$

$$y - y_1 = m(x - x_1)$$

$$y - (88) = -31(x - (-5))$$

$$y - 88 = -31x - 155$$

$$y = -31x - 67$$