# Basic Derivative Rules
## 2.3A – Product Rule

A: Use the Product Rule and Newton's Notation to find the derivative of each product.

#1) $f(x) = x^5 \cdot x^7$

$$f'(x) = (x^5)' \cdot x^7 + x^5 (x^7)'$$
$$= 5x^4 \cdot x^7 + x^5(7x^6)$$
$$= 5x^{11} + 7x^{11}$$
$$f'(x) = 12x^{11}$$

#2) $f(x) = x^5(x^3 - 1)$

$$f'(x) = (x^5)'(x^3-1) + x^5(x^3-1)'$$
$$= 5x^4(x^3-1) + x^5(3x^2)$$
$$= 5x^7 - 5x^4 + 3x^7$$
$$f'(x) = 8x^7 - 5x^4$$

#3) $f(x) = x^6(x^3 + 2x - 1)$

$$f'(x) = (x^6)'(x^3+2x-1) + x^6(x^3+2x-1)'$$
$$= 6x^5(x^3+2x-1) + x^6(3x^2+2)$$
$$= 6x^8 + 12x^6 - 6x^5 + 3x^8 + 2x^6$$
$$f'(x) = 9x^8 + 14x^6 - 6x^5$$

#4) $f(x) = x(5x^4 - 10)$

$$f'(x) = (x)'(5x^4-10) + x(5x^4-10)'$$
$$= (1)(5x^4-10) + x(20x^3)$$
$$= 5x^4 - 10 + 20x^4$$
$$f'(x) = 25x^4 - 10$$

#5) $f(x) = (x^2 + 1)(x^2 - 1)$

$$f'(x) = (x^2+1)'(x^2-1) + (x^2+1)(x^2-1)'$$
$$= 2x(x^2-1) + (x^2+1)(2x)$$
$$= 2x^3 - 2x + 2x^3 + 2x$$
$$f'(x) = 4x^3$$

#6) $f(x) = (x^5 - 1)(x^2 + 1)$

$$f'(x) = (x^5-1)'(x^2+1) + (x^5-1)(x^2+1)'$$
$$= 5x^4(x^2+1) + (x^5-1)(2x)$$
$$= 5x^6 + 5x^4 + 2x^6 - 2x$$
$$f'(x) = 7x^6 + 5x^4 - 2x$$

B: Use the Product Rule and Leibniz's Notation to find the derivative of each product.

#7) $f(x) = (x^2 + 2x)(7x + 4)$

$$\frac{d}{dx}f(x) = \frac{d}{dx}(x^2+2x)(7x+4) + (x^2+2x)\frac{d}{dx}(7x+4)$$
$$= (2x+2)(7x+4) + (x^2+2x)(7)$$
$$= 14x^2 + 14x + 8x + 8 + 7x^2 + 14x$$
$$\frac{df}{dx} = 21x^2 + 36x + 8$$

#8) $f(x) = x^{11}(x^2 - 5x + 11)$

$$\frac{df}{dx} = \frac{dx^{11}}{dx}(x^2-5x+11) + x^{11} \cdot \frac{d(x^2-5x+11)}{dx}$$
$$= 11x^{10}(x^2-5x+11) + x^{11}(2x-5)$$
$$= 11x^{12} - 55x^{11} + 121x^{10} + 2x^{12} - 5x^{11}$$
$$\frac{df}{dx} = 13x^{12} - 60x^{11} + 121x^{10}$$

#9) $f(x) = (\sqrt{x} - 1)(\sqrt{x} + 1)$

$$\frac{d}{dx}f(x) = \frac{d}{dx}(x^{\frac{1}{2}} - 1)(\sqrt{x}+1) + (\sqrt{x}-1)\frac{d}{dx}(x^{\frac{1}{2}}+1)$$
$$= \tfrac{1}{2}x^{-\frac{1}{2}}(\sqrt{x}+1) + (\sqrt{x}-1)(\tfrac{1}{2}x^{-\frac{1}{2}})$$
$$= \frac{\sqrt{x}+1}{2\sqrt{x}} + \frac{\sqrt{x}-1}{2\sqrt{x}}$$
$$= \frac{2\sqrt{x}}{2\sqrt{x}}$$
$$\frac{d}{dx}f(x) = 1$$

#10) $f(x) = (x^{1/3} - x^{1/5})(x^{1/3} + x^{1/5})$

$$\frac{d}{dx}f(x) = \frac{d}{dx}(x^{\frac{1}{3}} - x^{\frac{1}{5}}) \cdot (x^{\frac{1}{3}} + x^{\frac{1}{5}}) + (x^{\frac{1}{3}} - x^{\frac{1}{5}})\frac{d}{dx}(x^{\frac{1}{3}} + x^{\frac{1}{5}})$$
$$= (\tfrac{1}{3}x^{-\frac{2}{3}} - \tfrac{1}{5}x^{-\frac{4}{5}}) \cdot (x^{\frac{1}{3}} + x^{\frac{1}{5}}) + (x^{\frac{1}{3}} - x^{\frac{1}{5}})(\tfrac{1}{3}x^{-\frac{2}{3}} + \tfrac{1}{5}x^{-\frac{4}{5}})$$
$$= \tfrac{1}{3}x^{-\frac{1}{3}} + \tfrac{1}{3}x^{-\frac{2}{3}} \cdot x^{\frac{1}{5}} - \tfrac{1}{5}x^{-\frac{4}{5}} \cdot x^{\frac{1}{3}} - \tfrac{1}{5}x^{-\frac{3}{5}} + \tfrac{1}{3}x^{-\frac{1}{3}} + \tfrac{1}{5}x^{\frac{1}{3}} \cdot x^{-\frac{4}{5}} - \tfrac{1}{3}x^{\frac{1}{5}} \cdot x^{-\frac{2}{3}} - \tfrac{1}{5}x^{-\frac{3}{5}}$$
$$\frac{d}{dx}f(x) = \tfrac{2}{3}x^{-\frac{1}{3}} - \tfrac{2}{5}x^{-\frac{3}{5}}$$

Use the product rule and Leibniz.

#11) $f(x) = (x^4 + x^2 + 9)(x^3 - x)$

$\frac{d}{dx}f(x) = \frac{d}{dx}(x^4+x^2+9)\cdot(x^3-x) + (x^4+x^2+9)\frac{d}{dx}(x^3-x)$

$\boxed{\frac{d}{dx}f(x) = (4x^3+2x)(x^3-x) + (x^4+x^2+9)(3x^2-1)}$

(Tri)(Bi)

Don't bother

Use the product rule and Leibniz.

#12) $f(x) = (\sqrt{x} + 7)(\sqrt{x} + x^2)$

$\frac{d}{dx}f(x) = \frac{d}{dx}(x^{\frac{1}{2}}+7)\cdot(\sqrt{x}+x^2) + (\sqrt{x}+7)\frac{d}{dx}(x^{\frac{1}{2}}+x^2)$

$= \left(\frac{1}{2}x^{-\frac{1}{2}}\right)(\sqrt{x}+x^2) + (\sqrt{x}+7)\left(\frac{1}{2}x^{-\frac{1}{2}}+2x\right)$

$\boxed{\frac{d}{dx}f(x) = \frac{\sqrt{x}+x^2}{2\sqrt{x}} + (\sqrt{x}+7)\left(\frac{1}{2\sqrt{x}}+2x\right)}$

Good enough

**Flag Football**

#13) After playing flag football for *x* hours, a person's body temperature is $T(x) = x^3(2x - 5) + 98.6$ degrees Fahrenheit (for $0 \leq x \leq 3$). Find the rate of change after 2 hours.

| $T(x) = {}^\circ F$ | $x$ = hours | $T'(x) = {}^\circ F/\text{hours}$ |
|---|---|---|

$T'(x) = (x^3)'(2x-5) + x^3(2x-5)'$

$= 3x^2(2x-5) + x^3(2)$

$= 6x^3 - 15x^2 + 2x^3$

$T'(x) = 8x^3 - 15x^2$

$T'(2) = 8(2)^3 - 15(2)^2$

$= 8(8) - 15(4)$

$= 64 - 60$

$T'(2) = 4\ {}^\circ F/\text{hour}$

After two hours of playing flag football, a person's body temperature is increasing by 4 degrees per hour.

**iTunes Sales**

#14) After *x* weeks, weekly sales of a song on iTunes are expected to be $S(x) = 3x^2(10 - x^3)$ thousand for the first 2 weeks. Find the rate of change of sales after 2 weeks.

| $S(x)$ = songs in thousands | $x$ = week | $S'(x) = \frac{\text{songs (thousands)}}{\text{week}}$ |
|---|---|---|

$S'(x) = (3x^2)'(10-x^3) + 3x^2(10-x^3)'$

$= 6x(10-x^3) + 3x^2(-3x^2)$

$= 60x - 6x^4 - 9x^4$

$S'(x) = 60x - 15x^4$

$S'(2) = 60(2) - 15(2)^4$

$= 120 - 15(16)$

$= 120 - 240$

$S'(2) = -120$ thousand sales/week

After 2 weeks on iTunes, the weekly sales of a song are decreasing by 120,000 songs per week.