# Basic Derivative Rules
## 2.3 – Product Rule

### Product Rule

Newton's Notation

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

Ex A: Use the Product Rule and Newton's Notation to find the derivative of each product.

#1) $x^4 \cdot x^6$

$(x^4 \cdot x^6)' = (x^4)' x^6 + x^4 \cdot (x^6)'$

$= 4x^3 \cdot x^6 + x^4 \cdot 6x^5$

$= 4x^9 + 6x^9$

$(x^4 \cdot x^6)' = 10x^9$

#2) $y = (x^3 - x^2 + 7)(x^4 + 3)$

$y' = (x^3 - x^2 + 7)' \cdot (x^4 + 3) + (x^3 - x^2 + 7) \cdot (x^4 + 3)'$

$= (3x^2 - 2x)(x^4 + 3) + (x^3 - x^2 + 7)(4x^3)$

$= 3x^6 + 9x^2 - 2x^5 - 6x + 4x^6 - 4x^5 + 28x^3$

$y' = 7x^6 - 6x^5 + 28x^3 + 9x^2 - 6x$

### Product Rule

Leibniz's Notation

$$\frac{d}{dx}(f \cdot g) = \left(\frac{d}{dx} f\right) \cdot g + f \cdot \left(\frac{d}{dx} g\right)$$

Ex B: Use the Product Rule and Leibniz's Notation to find the derivative of each product.

#1) $x^4(4x^7 - 3x^2 + 12)$

$\frac{d}{dx}\left[x^4(4x^7 - 3x^2 + 12)\right]$

$= \frac{d}{dx} x^4 \cdot (4x^7 - 3x^2 + 12) + x^4 \cdot \frac{d}{dx}(4x^7 - 3x^2 + 12)$

$= 4x^3(4x^7 - 3x^2 + 12) + x^4(28x^6 - 6x)$

$= 16x^{10} - 12x^5 + 48x^3 + 28x^{10} - 6x^5$

$= 44x^{10} - 18x^5 + 48x^3$

#2) $y = \frac{15x+1}{x^3}$

$\frac{dy}{dx} = \frac{d}{dx}(15x+1) \cdot x^{-3} + (15x+1) \frac{d}{dx} x^{-3}$

$= 15 \cdot x^{-3} + (15x+1)(-3x^{-4})$

$= \frac{15}{x^3} + \frac{(15x+1)(-3)}{x^4}$

$= \frac{15 \cdot x}{x^3 \cdot x} + \frac{-45x - 3}{x^4}$

$= \frac{15x - 45x - 3}{x^4}$

$\frac{dy}{dx} = \frac{-30x - 3}{x^4}$

Ex C: Answer the following word problems.

**PS4 sales**

#1) After selling PS4s for $t$ weeks, the total sales are $S(t) = t^3(16 - t^2)$ thousand PS4s for the first 3 weeks of sales. Find the rate of change after week 2.

| $S(t)$ = thousand PS4s | $t$ = week | $S'(t)$ = thousand PS4s/week |
|---|---|---|

$S'(t) = (t^3)'(16 - t^2) + t^3(16 - t^2)'$

$= 3t^2(16 - t^2) + t^3(-2t)$

$= 48t^2 - 3t^4 - 2t^4$

$S'(t) = -5t^4 + 48t^2$

$S'(2) = -5(2)^4 + 48(2)^2$

$= -5(16) + 48(4)$

$= -80 + 192$

$S'(2) = 112$ thousand PS4s/week

After selling PS4s for 2 weeks, the total number of sales is increasing by 112,000 PS4s per week.

**Weeds**

#2) After pulling weeds for $t$ days, the total number of weeds in a flower garden can be represented by $W(t) = (t^2 + 1)(t^3 - 1)$ weeds. Find the rate of change after 8 days.

| $W(t)$ = weeds | $t$ = days | $W'(t)$ = weeds/day |
|---|---|---|

$W'(t) = (t^2+1)'(t^3-1) + (t^2+1)(t^3-1)'$

$= 2t(t^3-1) + (t^2+1)(3t^2)$

$= 2t^4 - 2t + 3t^4 + 3t^2$

$W'(t) = 5t^4 + 3t^2 - 2t$

$W'(8) = 5(8)^4 + 3(8)^2 - 2(8)$

$= 5(4096) + 3(64) - 16$

$= 20{,}480 + 192 - 16$

$W'(8) = 20{,}656$

After pulling weeds for 8 weeks, the number of weeds in the garden are STILL increasing by 20,656 weeds per week.