Basic Derivative Rules

2.4A – Quotient Rule

A: Find the derivative of each function using Newton's Notation.

#1) $y = \frac{x^5-1}{x^3} = x^2 - x^{-3}$

$y' = 2x + 3x^{-4}$

or

$y' = \frac{2x^5+3}{x^4}$

#2) $y = \frac{x-1}{x+1}$

$y' = \frac{(x-1)'(x+1) - (x-1)(x+1)'}{(x+1)^2}$

$y' = \frac{(1)(x+1) - (x-1)(1)}{(x+1)^2}$

$y' = \frac{x+1-x+1}{(x+1)^2}$

$y' = \frac{2}{(x+1)^2}$

#3) $y = \frac{3x^2+5}{x+7}$

$y' = \frac{(3x^2+5)'(x+7) - (3x^2+5)(x+7)'}{(x+7)^2}$

$= \frac{6x(x+7) - (3x^2+5)(1)}{(x+7)^2}$

$= \frac{6x^2+42x-3x^2-5}{(x+7)^2}$

$y' = \frac{3x^2+42x-5}{(x+7)^2}$

#4) $y = \frac{x^2-1}{x^2+1}$

$y' = \frac{(x^2-1)'(x^2+1) - (x^2-1)(x^2+1)'}{(x^2+1)^2}$

$= \frac{2x(x^2+1) - (x^2-1)(2x)}{(x^2+1)^2}$

$= \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}$

$y' = \frac{4x}{(x^2+1)^2}$

#5) $y = \frac{x^4-1}{x+1}$

$y' = \frac{(x^4-1)'(x+1) - (x^4-1)(x+1)'}{(x+1)^2}$

$= \frac{4x^3(x+1) - (x^4-1)(1)}{(x+1)^2}$

$= \frac{4x^4+4x^3-x^4+1}{(x+1)^2}$

$y' = \frac{3x^4+4x^3+1}{(x+1)^2}$

B: Find the derivative of each function using Leibniz's Notation.

#6) $y = \frac{x^2+3x-5}{x+1}$

$\frac{dy}{dx} = \frac{\frac{d}{dx}(x^2+3x-5)\cdot(x+1) - (x^2+3x-5)\frac{d}{dx}(x+1)}{(x+1)^2}$

$= \frac{(2x+3)(x+1) - (x^2+3x-5)(1)}{(x+1)^2}$

$= \frac{2x^2+5x+3-x^2-3x+5}{(x+1)^2}$

$\frac{dy}{dx} = \frac{x^2+2x+8}{(x+1)^2}$

#7) $y = \frac{x^3-2x^2}{x-2}$

$y = \frac{x^2(x-2)}{x-2}$

$y = x^2$

$\frac{dy}{dx} = 2x$

#8) $y = \frac{x^4+2x^2+1}{x^2+1}$

$y = \frac{(x^2+1)^2}{x^2+1}$

$y = x^2+1$

$\frac{dy}{dx} = 2x$

#9) $y = \frac{x^2-9}{x-3}$

$y = \frac{(x-3)(x+3)}{x-3}$

$y = x+3$

$\frac{dy}{dx} = 1$

Amp'd

#10) The number of bottles of Amp Energy drink that college students will buy in a month at a price of p dollars per bottle (for $p > \$0.50$) is $B(p) = \frac{100}{p+6}$. Find the rate of change of bottles purchased when the price is \$2 and interpret your answer.

$B(p)$ = bottles of Amp | p = \$ per bottle

$B'(p)$ = bottles / \$ per bottle

$$B'(p) = \frac{(100)'(p+6) - 100(p+6)'}{(p+6)^2}$$

$$= \frac{0(p+6) - 100(1)}{(p+6)^2}$$

$$B'(p) = \frac{-100}{(p+6)^2}$$

$$B'(2) = \frac{-100}{((2)+6)^2}$$

$$= \frac{-100}{(8)^2}$$

$$= \frac{-100}{64}$$

$B'(2) = -1.5625$ bottles/\$ per bottle

When Amp'd is being sold at \$2 per bottle, the number of sales per student per month will decrease by 1.5625 bottles per \$1 increase in price.

Or

When Amp'd is being sold at \$2 per bottle, if it's price is increased by \$1 it will result in a loss of sales of 1.5625 bottles per student per month.

EPA

#11) According to the EPA, the mpg of subcompact cars is $mpg(v) = \frac{-15v^2+1125v}{v^2-100v+3500}$ where v is the speed in miles per hour (for $35 \le v \le 65$).

a. Find $mpg'(v)$. *You don't need to simplify.*

b. Find $mpg'(45)$, $mpg'(55)$, $mpg'(65)$ using a calculator and interpret your answers.

c. What valuable lesson can be gained from the answers from part b?

a. $$mpg'(v) = \frac{(-15v^2+1125v)'(v^2-100v+3500) - (-15v^2+1125v)(v^2-100v+3500)'}{(v^2-100v+3500)^2}$$

$$mpg'(v) = \frac{(-30v+1125)(v^2-100v+3500) + (+15v^2-1125v)(2v-100)}{(v^2-100v+3500)^2}$$

b. $mpg'(45) \approx -.03$ mpg/mph

When traveling at 45 mph, you will lose .03 miles per gallon for each mph increase in speed.

$mpg'(55) = -.67$ mpg/mph

When traveling at 55 mph, you will lose .67 miles per gallon for each mph increase in speed.

$mpg'(65) = -.87$ mpg/mph

When traveling at 65 mph, you will lose .87 miles per gallon for each mph increase in speed.

c. Increasing your speed can greatly effect your mpg in a negative way.