Basic Derivative Rules
2.4 – Quotient Rule

Quotient Rule

Newton's Notation

$$\left(\frac{f}{g}\right)' = \frac{f'\cdot g - f\cdot g'}{g^2}$$

Ex A: Use the Quotient Rule and Newton's Notation.

#1) $\left(\frac{x^{10}}{x^4}\right)' = \frac{(x^{10})'\cdot x^4 - x^{10}(x^4)'}{(x^4)^2}$

$= \frac{10x^9\cdot x^4 - x^{10}\cdot 4x^3}{x^8}$

$= \frac{10x^{13} - 4x^{13}}{x^8}$

$= \frac{6x^{13}}{x^8}$

$\left(\frac{x^{10}}{x^4}\right)' = 6x^5$

#2) If $y = \frac{x^3}{x^2-4}$, then find y'.

$y' = \frac{(x^3)'(x^2-4) - x^3(x^2-4)'}{(x^2-4)^2}$

$y' = \frac{3x^2(x^2-4) - x^3(2x)}{(x^2-4)^2}$

$y' = \frac{3x^4 - 12x^2 - 2x^4}{(x^2-4)^2}$

$y' = \frac{x^4 - 12x^2}{(x^2-4)^2}$

Quotient Rule

Leibniz's Notation

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{\frac{d}{dx}(f)\cdot g - f\cdot\frac{d}{dx}(g)}{(g)^2}$$

Ex B: Use the Quotient Rule and Leibniz's Notation.

#1) If $y = \left(\frac{x^5-2}{x^3-1}\right)$, then find $\frac{dy}{dx}$.

$\frac{dy}{dx} = \frac{\frac{d}{dx}(x^5-2)\cdot(x^3-1) - (x^5-2)\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$

$= \frac{5x^4(x^3-1) - (x^5-2)(3x^2)}{(x^3-1)^2}$

$= \frac{5x^7 - 5x^4 - 3x^7 + 6x^2}{(x^3-1)^2}$

$\frac{dy}{dx} = \frac{2x^7 - 5x^4 + 6x^2}{(x^3-1)^2}$

#2) $\frac{d}{dx}\left(\frac{x^3+x}{x^3-1}\right) = \frac{\frac{d}{dx}(x^3+x)\cdot(x^3-1) - (x^3+x)\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$

$= \frac{(3x^2+1)(x^3-1) - (x^3+x)(3x^2)}{(x^3-1)^2}$

$= \frac{\cancel{3x^5} + x^3 - 3x^2 - 1 - \cancel{3x^5} - 3x^3}{(x^3-1)^2}$

$\frac{dy}{dx} = \frac{-2x^3 - 3x^2 - 1}{(x^3-1)^2}$

$C(x) = $ \$ per gallon | $x = $ % of purity

$C'(x) = \frac{\$ \text{ gallon}}{\% \text{ of purity}}$

Ex C: Answer the following word problems.

Drinking Water

Gnadenhutten must purify its drinking water. If the cost of purifying a gallon of water to a purity of x percent is $C(x) = \frac{2}{100-x}$ dollars for $80 < x < 100$, find the rate of change of the purification costs when the purity is 92% and 98% and interpret your answer.

$$C'(x) = \frac{(2)'(100-x) - 2\cdot(100-x)'}{(100-x)^2}$$

$$= \frac{0(100-x) - 2(-1)}{(100-x)^2}$$

$$C'(x) = \frac{2}{(100-x)^2}$$

$$C'(92) = \frac{2}{(100-92)^2} = \frac{2}{(8)^2} = \frac{2}{64} = \frac{1}{32} = \$.03125 \frac{\text{per gallon}}{\% \text{ purity}}$$

$$C'(98) = \frac{2}{(100-98)^2} = \frac{2}{(2)^2} = \frac{2}{4} = \$.50 \frac{\text{per gallon}}{\% \text{ purity}}$$

At 92% purity, it will cost a little over 3 cents per gallon of water to increase the purity by 1%.

At 98% purity, it will cost a little over 50 cents per gallon of water to increase the purity by 1%.