Basic Derivative Rules
2.4B – Product & Quotient Rules

Find the derivative of each function.

#1) $y = (x^2+1)\frac{x-1}{x+1}$

$y' = (x^2+1)'\left(\frac{x-1}{x+1}\right) + (x^2+1)\left(\frac{x-1}{x+1}\right)'$

$= 2x\left(\frac{x-1}{x+1}\right) + (x^2+1)\frac{(x-1)'(x+1) - (x-1)(x+1)'}{(x+1)^2}$

$= 2x\left(\frac{x-1}{x+1}\right) + (x^2+1)\frac{1\cdot(x+1) - (x-1)\cdot 1}{(x+1)^2}$

$= 2x\left(\frac{x-1}{x+1}\right) + (x^2+1)\frac{x+1-x+1}{(x+1)^2}$

$= 2x\left(\frac{x-1}{x+1}\right) + (x^2+1)\frac{2}{(x+1)^2}$

$= \frac{2x(x-1)}{x+1} + \frac{2(x^2+1)}{(x+1)^2}$

$= \frac{2x(x-1)(x+1)}{(x+1)^2} + \frac{2x^2+2}{(x+1)^2}$

$= \frac{2x(x^2-1)}{(x+1)^2} + \frac{2x^2+2}{(x+1)^2}$

$= \frac{2x^3-2x+2x^2+2}{(x+1)^2}$

$y' = \frac{2x^3+2x^2-2x+2}{(x+1)^2}$

#2) $y = (x+1)\frac{x^2-x}{x+1}$

$y = x^2 - x$

$y' = 2x - 1$

#3) $y = \frac{(x-1)(x^2-12)}{x^3+1}$

$$y' = \frac{[(x-1)(x^2-12)]'(x^3+1) - (x-1)(x^2-12)(x^3+1)'}{(x^3+1)^2}$$

$$= \frac{[(x-1)'(x^2-12) + (x-1)(x^2-12)'](x^3+1) - (x-1)(x^2-12)(3x^2)}{(x^3+1)^2}$$

$$= \frac{[1\cdot(x^2-12) + (x-1)(2x)](x^3+1) - (x-1)(x^2-12)(3x^2)}{(x^3+1)^2}$$

$$= \frac{[x^2-12+2x^2-2x](x^3+1) - 3x^2(x-1)(x^2-12)}{(x^3+1)^2}$$

$$y' = \frac{(3x^2-2x-12)(x^3+1) - 3x^2(x-1)(x^2-12)}{(x^3+1)^2}$$

#4) $y = \frac{\sqrt{x}-2}{\sqrt{x}+2}$

$$y' = \frac{(x^{\frac{1}{2}}-2)'(\sqrt{x}+2) - (\sqrt{x}-2)(x^{\frac{1}{2}}+2)'}{(\sqrt{x}+2)^2}$$

$$= \frac{\left(\frac{1}{2}x^{-\frac{1}{2}}\right)(\sqrt{x}+2) - (\sqrt{x}-2)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{(\sqrt{x}+2)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}+2) - (\sqrt{x}-2)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}+2)^2}$$

$$= \frac{\frac{1}{2} + \frac{1}{\sqrt{x}} - \frac{1}{2} + \frac{1}{\sqrt{x}}}{(\sqrt{x}+2)^2}$$

$$= \frac{\frac{2}{\sqrt{x}}}{(\sqrt{x}+2)^2}$$

$$= \frac{2}{\sqrt{x}} \cdot \frac{1}{(\sqrt{x}+2)^2}$$

$$y' = \frac{2}{\sqrt{x}(\sqrt{x}+2)^2}$$