Basic Derivative Rules

2.5A – Differentiating $e^{f(x)}$ and $\ln f(x)$

A: Find the derivative of each function.

#1) $f(x) = (x^2 + x)\ln(x)$

$f'(x) = (x^2+x)' \cdot \ln(x) + (x^2+x)[\ln(x)]'$

$= (2x+1)\ln(x) + (x^2+x)\frac{1}{x}$

$f'(x) = (2x+1)\ln(x) + x+1$

#2) $f(x) = \ln(\sqrt{x})$

$f'(x) = \frac{(\sqrt{x})'}{\sqrt{x}}$

$= \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{x}}$

$= \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}}$

$f'(x) = \frac{1}{2x}$

#3) $f(x) = \ln(x^3)$

$f'(x) = \frac{(x^3)'}{x^3}$

$= \frac{3x^2}{x^3}$

$f'(x) = \frac{3}{x}$

#4) $f(x) = x\ln(x)$

$f'(x) = x' \cdot \ln(x) + x \cdot [\ln(x)]'$

$= 1 \cdot \ln(x) + x \cdot \frac{1}{x}$

$f'(x) = \ln(x) + 1$

#5) $f(x) = \frac{\ln(x)}{x}$

$\frac{df}{dx} = \frac{\frac{d}{dx}\ln(x) \cdot x - \ln(x) \cdot \frac{d}{dx}x}{x^2}$

$\frac{df}{dx} = \frac{\frac{1}{x} \cdot x - \ln(x) \cdot 1}{x^2}$

$\frac{df}{dx} = \frac{1 - \ln(x)}{x^2}$

#6) $f(x) = e^x$

$\frac{df}{dx} = x' \cdot e^x$

$= 1 \cdot e^x$

$\frac{df}{dx} = e^x$

#7) $f(x) = e^{3x^2+9x-1}$

$\frac{d}{dx}f(x) = (3x^2+9x-1)' e^{3x^2+9x-1}$

$\frac{df}{dx} = (6x+9)e^{3x^2+9x-1}$

#8) $f(x) = e^{-x}$

$f'(x) = (-x)' \cdot e^{-x}$

$= -1 \cdot e^{-x}$

$f'(x) = -e^{-x}$

Basic Derivative Rules

2.5A – Differentiating $e^{f(x)}$ and $\ln f(x)$

#9) $f(x) = e^{\ln(x)}$

$f(x) = x$

$f'(x) = 1$

#10) $f(x) = \ln(e^{x^2})$

$f'(x) = \frac{(e^{x^2})'}{e^{x^2}}$

$= \frac{2x \cdot e^{x^2}}{e^{x^2}}$

$f'(x) = 2x$

#11) $f(x) = x^e$

$\frac{df}{dx} = ex^{e-1}$

#12) $f(x) = ex$

$\frac{df}{dx} = e$

#13) $f(x) = \frac{x}{\ln(x)}$

$f'(x) = \frac{x' \cdot \ln(x) - x \cdot [\ln(x)]'}{[\ln(x)]^2}$

$= \frac{1 \cdot \ln(x) - x \cdot \frac{1}{x}}{\ln^2(x)}$

$f'(x) = \frac{\ln(x) - 1}{\ln^2(x)}$

#14) $f(x) = e^{21}$

$\frac{df}{dx} = 0$

#15) $f(x) = e^{3x} - x\ln(x) + 4x^2 + 1$

$f'(x) = 3e^{3x} - [x' \cdot \ln(x) + x \cdot [\ln(x)]'] + 8x$

$= 3e^{3x} - [1 \cdot \ln(x) + x \cdot \frac{1}{x}] + 8x$

$f'(x) = 3e^{3x} - \ln x - 1 + 8x$

#16) $f(x) = e\ln(x)$

$f'(x) = e \cdot \frac{1}{x}$

$f'(x) = \frac{e}{x}$

Basic Derivative Rules
2.5A – Differentiating $e^{f(x)}$ and $\ln f(x)$

B: Evaluate each derivative

#17) $f(x) = x^3 \ln(x)$, find $f'(e)$

$$f'(x) = (x^3)' \cdot \ln(x) + x^3 \cdot [\ln(x)]'$$
$$= 3x^2 \cdot \ln(x) + x^3 \cdot \frac{1}{x}$$
$$f'(x) = 3x^2 \ln(x) + x^2$$
$$f'(e) = 3(e)^2 \ln(e) + (e)^2$$
$$= 3e^2 \cdot 1 + e^2$$
$$= 3e^2 + e^2$$
$$f'(e) = 4e^2$$

#18) $\frac{d}{dx}\left(e^{x^4+4}\right)\Big|_{x=1}$ $= 4x^3 e^{x^4+4}\Big|_{x=1}$

$$= 4(1)^3 e^{(1)^4+4}$$
$$= 4(1) e^{1+4}$$
$$\frac{d}{dx}\left(e^{x^4+4}\right)\Big|_{x=1} = 4e^5$$

#19) $f(x) = x^2 \ln(x) - x$, find $f'(1)$

$$f'(x) = (x^2)' \ln(x) + x^2 [\ln(x)]' - 1$$
$$= 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} - 1$$
$$f'(x) = 2x \ln(x) + x - 1$$
$$f'(1) = 2(1)\ln(1) + (1) - 1$$
$$= 2 \cdot 0 + 0$$
$$= 0 + 0$$
$$f'(1) = 0$$

#20) $\frac{d}{dx}\left(e^{\sqrt{x}}\right)\Big|_{x=1}$ $= (\sqrt{x})' e^{\sqrt{x}}\Big|_{x=1}$

$$= \frac{1}{2\sqrt{x}} e^{\sqrt{x}}\Big|_{x=1}$$
$$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}\Big|_{x=1}$$
$$= \frac{e^{\sqrt{1}}}{2\sqrt{1}}$$
$$\frac{d}{dx}\left(e^{\sqrt{x}}\right)\Big|_{x=1} = \frac{e}{2}$$

Investment

#21) A sum of $1000 at 5% interest compounded continuously will grow to $V(t) = 1000e^{0.05t}$ dollars in t years. Find the rate of growth after:

a. 0 years
b. 10 years

$V'(t) = 1000(0.05t)' \cdot e^{0.05t}$

$= 1000(0.05)e^{0.05t}$

$V'(t) = 50e^{0.05t}$

a. $V'(0) = 50e^{0.05(0)}$
$= 50e^0$
$= 50(1)$
$V'(0) = \$50/\text{year}$

b. $V'(10) = 50e^{0.05(10)}$
$= 50e^{0.5}$
$= \$82.44/\text{year}$

At the initial time of investment, the money is growing by $50 per year.

In 10 years, the money is growing by $82.44 per year.

Depreciation

#22) A $30,000 automobile depreciates so that its value after t years is $V(t) = 30{,}000e^{-0.35t}$ dollars. Find the rate of change of its value …

a. when it is brand spanking new
b. after 2 years

$V'(t) = (-0.35t)' \cdot 30{,}000e^{-0.35t}$

$= (-0.35) \cdot 30{,}000e^{-0.35t}$

$V'(t) = -10{,}500e^{-0.35t}$

a. $V'(0) = -10{,}500e^{-.35(0)}$
$= -10{,}500e^0$
$V'(0) = -\$10{,}500/\text{year}$

b. $V'(2) = -10{,}500e^{-.35(2)}$
$= -10{,}500e^{-0.70}$
$V'(2) \approx -\$5214.15/\text{year}$

The moment you buy a car, it is depreciating by $10,500 per year.

Two years after purchase, the car is depreciating by $5,214.15 per year.

Candle Sticks

#23) If $D(p) = 1000e^{-0.01p}$ is the consumer demand for George's homemade candle sticks (which he advertises as "imported from the best Italian ears") and p is the selling price in dollars, find $D'(100)$ and interpret your answer.

$D(p)$ = candlesticks sold	p = price	$D' = \frac{\text{candlesticks}}{\text{dollar}}$

$D'(p) = 1000(-0.01p)'e^{-0.01p}$

$= 1000(-0.01)e^{-0.01p}$

$D'(p) = -10e^{-0.01p}$

$D'(100) = -10e^{-.01(100)}$

$= -10e^{-1}$

$= \frac{-10}{e}$

$D'(100) \approx -37$ candlesticks/dollar

When the candlesticks cost $100, increasing the price by $1 will result in 37 fewer sales.

Forever Burning Matches ®

#24) If $D(p) = 4000e^{-0.02p}$ is the consumer demand for George's Forever Burning Matches ® and p is the selling price in dollars, find $D'(50)$ and interpret your answer.

$D(p)$ = matches sold	p = price	$D' = \frac{\text{matches}}{\text{dollar}}$

$D'(p) = 4000(-0.02p)'e^{-0.02p}$

$= 4000(-0.02)e^{-0.02p}$

$D'(p) = -20e^{-0.02p}$

$D'(50) = -20e^{-0.02(50)}$

$= -20e^{-1}$

$= \frac{-20}{e}$

$D'(50) \approx -7$ matches/dollar

When the matches cost $50, increasing the price by $1 will result in 7 fewer sales.