Basic Derivative Rules
2.5 – Differentiating $e^{f(x)}$ and $\ln f(x)$

Derivative of $\ln[f(x)]$

$$\frac{d}{dx}[\ln(f)] = \frac{\frac{d}{dx}(f)}{f}$$

Ex A: Find the derivative of each function.

#1) $f(x) = \ln(x^2 - x + 6)$

$$f'(x) = \frac{(x^2-x+6)'}{x^2-x+6}$$

$$f'(x) = \frac{2x-1}{x^2-x+6}$$

#2) $f(x) = \ln(x^4 + 9x^2 - 8)$

$$\frac{df}{dx} = \frac{\frac{d}{dx}(x^4+9x^2-8)}{x^4+9x^2-8}$$

$$\frac{df}{dx} = \frac{4x^3+18x}{x^4+9x^2-8}$$

#3) $f(x) = \ln(x^2 - 1)^5$

$$f'(x) = \frac{[(x^2-1)^5]'}{(x^2-1)^5} \longrightarrow \text{CAN'T DO.}$$

Instead

$$f(x) = \ln(x^2-1)^5$$

$$f(x) = 5\ln(x^2-1)$$

$$f'(x) = 5\,\frac{(x^2-1)'}{x^2-1}$$

$$= 5\,\frac{2x}{x^2-1}$$

$$f'(x) = \frac{10x}{x^2-1}$$

#4) $f(x) = x^2\ln(x)$

$$\frac{df}{dx} = \frac{d}{dx}x^2\cdot\ln(x) + x^2\cdot\frac{d}{dx}\ln(x)$$

$$\frac{df}{dx} = 2x\cdot\ln(x) + x^2\cdot\frac{x'}{x}$$

$$\frac{df}{dx} = 2x\cdot\ln(x) + x^2\cdot\frac{1}{x}$$

$$\frac{df}{dx} = 2x\ln(x) + x$$

#5) $f(x) = \frac{\ln(x)}{x}$

$$f'(x) = \frac{[\ln(x)]'\cdot x - \ln(x)\cdot x'}{x^2}$$

$$= \frac{\frac{x'}{x}\cdot x - \ln(x)\cdot 1}{x^2}$$

$$= \frac{\frac{1}{x}\cdot x - \ln(x)}{x^2}$$

$$f'(x) = \frac{1-\ln(x)}{x^2}$$

Derivative of $e^{f(x)}$

$$\frac{d}{dx}(e^f) = \frac{d}{dx}(f)\cdot e^f$$

Ex B: Find each derivative.

#1) $\frac{d}{dx}e^{x^2+x-1} = \frac{d}{dx}(x^2+x-1)\cdot e^{x^2+x-1}$

$$= (2x+1)e^{x^2+x-1}$$

#2) $\frac{d}{dx}e^{\frac{x^2}{2}} = \frac{d}{dx}\left(\frac{1}{2}x^2\right)\cdot e^{\frac{x^2}{2}}$

$$= x e^{\frac{x^2}{2}}$$

#3) $\left(e^{\frac{1}{4}x^4-1}\right)' = \left(\frac{1}{4}x^4-1\right)' e^{\frac{1}{4}x^4-1}$

$$= x^3 e^{\frac{1}{4}x^4-1}$$

#4) $\left(\frac{e^x}{x}\right)' = \frac{(e^x)'\cdot x - e^x\cdot x'}{x^2}$

$$= \frac{x'\cdot e^x\cdot x - e^x(1)}{x^2}$$

$$= \frac{1\cdot e^x\cdot x - e^x}{x^2}$$

$$\left(\frac{e^x}{x}\right)' = \frac{xe^x - e^x}{x^2}$$

#5) If $f(x) = xe^x$, find $f'(1)$

$$f'(x) = x'e^x + x(e^x)'$$

$$= 1\cdot e^x + xe^x$$

$$f'(x) = e^x + xe^x$$

$$f'(1) = e^{(1)} + (1)e^{(1)}$$

$$f'(1) = e + e$$

$$f'(1) = 2e$$

#6) If $f(x) = e^x\ln(x)$, find $f'(1)$

$$f'(x) = (e^x)'\ln(x) + e^x[\ln(x)]'$$

$$= e^x\cdot\ln(x) + e^x\cdot\frac{1}{x}$$

$$f'(x) = e^x\ln(x) + \frac{e^x}{x}$$

#7) $[\ln(x^2 + e^x)]' = \frac{(x^2+e^x)'}{x^2+e^x}$

$$= \frac{2x+e^x}{x^2+e^x}$$

POLE VAULTING

#1) After t weeks of practice a pole vaulter can vault $H(t) = 14(1 - e^{-0.10t})$ feet. Find the rate of change of the athlete's jumps after

a. 0 weeks (at the beginning of training)

b. 12 weeks of training

H(t)= feet	t= week	H'(t)= ft/week

$H(t) = 14 - 14e^{-0.10t}$

$H'(t) = (-.10t)'(-14e^{-0.10t})$

$= (-.10)(-14e^{-0.10t})$

$H'(t) = 1.4e^{-0.10t}$

a. $H'(0) = 1.4e^{-0.10(0)}$

$H'(0) = 1.4e^{0}$

$H'(0) = 1.4(1)$

$H'(0) = 1.4$ ft/week

After no practice, a pole vaulter can increase his jumping by 1.4 feet per week.

b. $H'(12) = 1.4e^{-0.10(12)}$

$= 1.4e^{-1.2}$

$H'(12) = .42$ ft/week

After 12 weeks of practice, a pole vaulter can increase his jumping by 0.42 feet per week.