Derivative Applications
3.1A – Marginal & Other Applications

We Got Portals Co.

#1) We Got Portals Company finds that its cost function is $C(x) = 60{,}000\sqrt{x} - 4000\sqrt[3]{x}$ dollars, where x is the daily production of magical portals.

a. Find the marginal cost function.
b. Find the marginal cost when 8 portals have been produced.
c. Interpret your answer from part b.

a.

$C(x) = \$\text{cost}$
$x = \text{portals}$
$C'(x) = \$/\text{portal}$

$C(x) = 60{,}000x^{\frac{1}{2}} - 4000x^{\frac{1}{3}}$

$MC(x) = C'(x) = 30{,}000x^{-\frac{1}{2}} - \frac{4000}{3}x^{-2/3}$

$MC(x) = \frac{30{,}000}{\sqrt{x}} - \frac{4000}{3(\sqrt[3]{x})^2}$

b.

$MC(8) = \frac{30000}{\sqrt{8}} - \frac{4000}{3(\sqrt[3]{8})^2}$

$= \frac{30{,}000}{\sqrt{8}} - \frac{\cancel{4}000}{3 \cdot \cancel{4}}$

$MC(8) = \frac{30{,}000}{\sqrt{8}} - \frac{1000}{3}$

$MC(8) = \$10{,}273.27/\text{portal}$

c.

When 8 portals have been produced, the total cost is increasing by \$10.273.27 per portal produced.

or

When 8 portals have been produced, the cost to produce the next portal is \$10,273.27.

Portal Remover Inc.

#2) Portal Remover Inc. finds that its revenue function is $R(x) = 3000\sqrt[3]{x} + 64\sqrt{x}$ dollars, where x is the daily sales of portal removers.

a. Find the marginal revenue function.
b. Find the marginal revenue when 64 portal removers have been sold.
c. Interpret your answer from part b.

a.

$R(x) = \$\text{Revenue}$
$x = \text{portal removers}$
$R'(x) = \$/\text{remover}$

$R(x) = 3000x^{1/3} + 64x^{1/2}$

$MR(x) = R'(x) = 1000x^{-2/3} + 32x^{-\frac{1}{2}}$

$MR(x) = \frac{1000}{(\sqrt[3]{x})^2} + \frac{32}{\sqrt{x}}$

b.

$MR(64) = \frac{1000}{(\sqrt[3]{64})^2} + \frac{32}{\sqrt{64}}$

$= \frac{1000}{16} + \frac{32}{8}$

$= 62.5 + 4$

$MR(64) = \$66.5/\text{Remover}$

c.

When 64 portal removers have been sold, the total revenue is increasing by \$66.50 per portal remover sold.

or

When 64 portal removers have been sold, the revenue from the next portal sale will be \$66.50.

Portal Med Supply

#3) Portal Med Supply find that its total profit from selling x vomit bags is $P(x) = 0.02x^{3/2} - 1500$ dollars.

a. Find Portal Med Supply's marginal profit function.
b. Find the marginal profit when 5,000 vomit bags have been sold.
c. Interpret your answer from part b.

a.

$P(x) = \$\text{profit}$
$x = \text{vomit bags}$
$P'(x) = \$/\text{bag}$

$MP(x) = P'(x) = 0.03x^{\frac{1}{2}}$

$MP(x) = 0.03x^{\frac{1}{2}}$

b.

$MP(5000) = 0.03\sqrt{5000}$
$= \$2.12/\text{bag}$

c.

When 5000 portal vomit bags have been sold, the total profit is increasing by \$2.12 per vomit bag sold.

OR

When 5000 portal vomit bags are sold, the profit from selling the next bag is \$2.12.

Portal Research and Development Labs

#4) Portal Research and Development Labs finds that the population of a city will be $P(x) = 12{,}000 - 12x + 6000x^2 + 10x^{-3}$ people x years after portal technology enters the city.

a. Find the rate of change of population x years after portal tech enters the city.
b. Find the rate of change 2 years from now.
c. Interpret your answer from part b.
d. Find the rate of change 10 years from now.
e. Interpret your answer from part d.

$P(x) = \text{people}$
$x = \text{years}$
$P'(x) = \text{peeps}/\text{year}$

a.

$P'(x) = -12 + 12{,}000x - 30x^{-4}$

b.

$P'(2) = -12 + 12000(2) - \frac{30}{(2)^4}$
$P'(2) = -12 + 24{,}000 - \frac{30}{16}$
$P'(2) = 23{,}988 - \frac{30}{16}$
$P'(2) \approx 23{,}986 \text{ people}/\text{year}$

c.

Two years after portal tech enters a city, the population is growing by 23,986 people per year.

d. $P'(10) = -12 + 12{,}000(10) - \frac{30}{(10)^4}$
$P'(10) = -12 + 120{,}000 - \frac{30}{1000}$
$P'(10) = 119988 - \frac{30}{1000}$
$P'(10) \approx 119{,}988 \text{ people}/\text{year}$

e.

Ten years after portal tech enters a city, the population is growing by 119,988 people per year.

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Turtle Flu

#5) The number of Mario Brothers that have been newly infected on day t of a turtle flu epidemic is $f(t) = 25t^2 - 3t^3$ for $0 \le t \le 5$.

a. Find the number of newly infected Brothers on day 2.
b. Interpret your answer from part a.
c. Find the instantaneous rate of change on day 2.
d. Interpret your answer from part c.

a.

f(t) = mario Bros (newly infected)
t = days of flu epidemic

$f(2) = 25(2)^2 - 3(2)^3$
$= 25(4) - 3(8)$
$= 100 - 24$
$f(2) = 76$ mario Bros (newly infected)

b. On day 2 of a turtle flu epidemic, the number of newly infected Mario Bros is 76.

c.

$f'(t) = 25t - 9t^2$
$f'(2) = 25(2) - 9(2)^2$
$f'(2) = 50 - 9(4)$
$f'(2) = 50 - 36$
$f'(2) = 14$ mario Bros / day

d. On day 2 of a turtle flu epidemic, the number of newly infected Mario Bros is increasing by 14 Mario Bros per day.

Turtle Classifieds

#6) It has been estimated that the total number of turtles who will see a Craigslist add that has run for d consecutive days is $N(d) = 10{,}000 - \frac{5{,}000}{d}$ turtles.

a. Find $N(5)$.
b. Interpret your answer from part a.
c. Find $N'(5)$.
d. Interpret your answer form part c.

a.

N(d) = total turtles
d = days (consecutive add days)

$N(5) = 10{,}000 - \frac{5{,}000}{(5)}$
$= 10{,}000 - 1{,}000$
$N(5) = 9{,}000$ turtles

b. After an ad has been on Craigslist for 5 days, the total number of turtles who have seen the ad is 9000.

c.

$N(d) = 10{,}000 - 5000d^{-1}$
$N'(d) = 5000d^{-2}$
$N'(d) = \frac{5000}{d^2}$
$N'(5) = \frac{5000}{(5)^2}$
$N'(5) = \frac{5000}{25}$
$N'(5) = 200$ turtles/day

d. After an ad has been on Craigslist for 5 days, the total number of turtles who have seen the ad is increasing by 200 turtles per day.

Turtle Tech

#7) Turtle Tech finds that a turtle can memorize $I(t) = 36\sqrt{t}$ Italian phrases after being stomped t times by a plumber for $0 \le t \le 14$.

a. Find the instantaneous rate of change of the phrases.
b. Find the instantaneous rate of change after 4 stomps.
c. Interpret your answer from part b.

a.

$I(t)$ = Italian phrases memorized
t = stomps by plumber

$I(t) = 36t^{\frac{1}{2}}$

$I'(t) = 18t^{-\frac{1}{2}}$

$I'(t) = \frac{18}{\sqrt{t}}$

b.

$I(4) = \frac{18}{\sqrt{(4)}}$

$= \frac{18}{2}$

$I(4) = 9$ (Italian phrases) / stomp

c.

After 4 stomps in the head by a plumber, the number of Italian phrases a turtle can memorize is increasing by 9 Italian phrases per stomp.

Turtle Chemical Plant

#8) Turtle Chemical Plant burns oil and as a result the amount sulfur dioxide pollution blowing x miles downwind of the plant is $s(x) = 59x^{-2}$ parts per minute.

a. Find $s(2)$.
b. Interpret your answer from part a.
c. Find $s'(2)$.
d. Interpret your answer from part c.

a.

$S(x)$ = parts per minute (sulfur dioxide pollution)
x = miles downwind

$S(x) = \frac{59}{x^2}$

$S(2) = \frac{59}{(2)^2}$

$= \frac{59}{4}$

$S(2) = 14.75$ parts per minute

b.

Two miles downwind from Turtle Chemical Plant, the amount of sulfur dioxide pollution is 14.75 parts per minute.

c.

$S(x) = 59x^{-2}$

$S'(x) = -118x^{-3}$

$S'(x) = \frac{-118}{x^3}$

$S'(2) = \frac{-118}{(2)^3}$

$= \frac{-118}{8}$

$S'(2) = -14.75$ $\frac{\text{parts per minute}}{\text{mile}}$

d.

Two miles downwind from Turtle Chemical Plant, the amount of sulfur dioxide pollution is decreasing by 14.75 parts per minute per mile.