Derivative Applications 3.1 – Marginal & Other Applications

There is another interpretation for the derivative, one that is particularly important in business and economics. Suppose that a company has calculated its *revenue, cost,* and *profit functions* as defined below.

If x = quantity produced or sold, then the following equations are true.

Cost Function C(x) = (Total cost of producing x units)C(x) = (unit cost)x + (fixed cost)

Revenue Function R(x) = (Total revenue (income) from selling x units)R(x) = (selling price per unit)x

Profit Function P(x) = (Total profit from producing & selling x units)P(x) = R(x) - C(x) The term *marginal cost* means the cost of producing one more unit. The cost of one more unit is the *rate* at which total costs are rising (measured in dollars per unit.) If we calculate rates of change as *instantaneous* rates of change (that is, derivatives), we see that the marginal cost function MC(x) is the derivative of the cost function:

The term *marginal revenue*, MR(x), means the revenue from selling one more unit.

The term *marginal profit*, MP(x), means the profit from selling one more unit.

Marginal Cost Function MC(x) = C'(x) Marginal cost is the derivative of cost

Marginal Revenue Function MR(x) = R'(x) Marginal revenue is the derivative of revenue

Marginal Profit Function MP(x) = P'(x) Marginal profit is the derivative of profit

Derivative: slope, instantaneous rates of change, marginal.

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Ex A: Finding and Interpreting Marginal Cost

A company manufactures cordless telephones and finds that its cost function (the total cost of manufacturing *x* telephones) is $C(x) = 400\sqrt{x} + 500$ dollars, where *x* is the number of telephones produced.

a. Find the marginal cost function MC(x).

$$C(x) = \frac{1}{4} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6}$$

$$C'(x) = \frac{1}{6} \frac{1$$

b. Find the marginal cost when 100 telephones have been produced.

$$M((100)) = \frac{200}{\sqrt{100}}$$

= $\frac{200}{10}$
= $\frac{200}{10}$

c. Interpret your answer.

When 100 phones have been produced, the total cost is increasing by \$20 per phone.

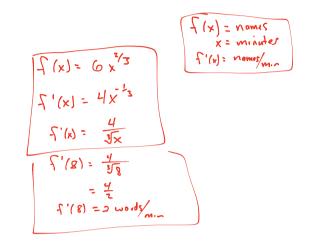
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When IOO phones have been produced, it costs \$20 to make the next phone.

Ex B: Finding a Learning Rate

A psychology researcher finds that the number of names that a person can memorize in x minutes is approximately $f(x) = 6\sqrt[3]{x^2}$.

a. Find the instantaneous rate of change of this function in 8 minutes.



b. Interpret your answer.

Eight minutes after memorizing began, the total words memorized is increasing by 2 words per minute.