Derivative Applications

3.1 – Marginal & Other Applications

There is another interpretation for the derivative, one that is particularly important in business and economics. Suppose that a company has calculated its *revenue, cost,* and *profit functions* as defined below.

If x = *quantity produced or sold*, then the following equations are true.

Cost Function
$C(x)$ = (Total cost of producing x units)
$C(x) = (unit\ cost)x + (fixed\ cost)$

Revenue Function
$R(x)$ = (Total revenue (income) from selling x units)
$R(x) = (selling\ price\ per\ unit)x$

Profit Function
$P(x)$ = (Total profit from producing & selling x units)
$P(x) = R(x) - C(x)$

The term *marginal cost* means the cost of producing one more unit. The cost of one more unit is the *rate* at which total costs are rising (measured in dollars per unit.) If we calculate rates of change as *instantaneous* rates of change (that is, derivatives), we see that the marginal cost function $MC(x)$ is the derivative of the cost function:

The term *marginal revenue*, $MR(x)$, means the revenue from selling one more unit.

The term *marginal profit*, $MP(x)$, means the profit from selling one more unit.

Marginal Cost Function
$MC(x) = C'(x)$ *Marginal cost is the derivative of cost*

Marginal Revenue Function
$MR(x) = R'(x)$ *Marginal revenue is the derivative of revenue*

Marginal Profit Function
$MP(x) = P'(x)$ *Marginal profit is the derivative of profit*

Derivative: slope, instantaneous rates of change, marginal.

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Ex A: Finding and Interpreting Marginal Cost

A company manufactures cordless telephones and finds that its cost function (the total cost of manufacturing x telephones) is $C(x) = 400\sqrt{x} + 500$ dollars, where x is the number of telephones produced.

a. Find the marginal cost function $MC(x)$.

$\boxed{\begin{array}{l} C(x) = \text{total cost } \$ \\ x = \#\text{ of phones} \\ C'(x) = \$/\text{phone} \end{array}}$

$$C(x) = 400x^{\frac{1}{2}} + 500$$

$$MC = C'(x) = 200x^{-\frac{1}{2}}$$

$$MC(x) = \frac{200}{\sqrt{x}}$$

b. Find the marginal cost when 100 telephones have been produced.

$$MC(100) = \frac{200}{\sqrt{(100)}}$$

$$= \frac{200}{10}$$

$$= \$20/\text{phone}$$

c. Interpret your answer.

When 100 phones have been produced, the total cost is increasing by \$20 per phone.

or

When 100 phones have been produced, it costs \$20 to make the next phone.

Ex B: Finding a Learning Rate

A psychology researcher finds that the number of names that a person can memorize in x minutes is approximately $f(x) = 6\sqrt[3]{x^2}$.

a. Find the instantaneous rate of change of this function in 8 minutes.

$\boxed{\begin{array}{l} f(x) = \text{names} \\ x = \text{minutes} \\ f'(x) = \text{names}/\text{min} \end{array}}$

$\boxed{\begin{array}{l} f(x) = 6x^{2/3} \\ f'(x) = 4x^{-1/3} \\ f'(x) = \dfrac{4}{\sqrt[3]{x}} \end{array}}$

$\boxed{\begin{array}{l} f'(8) = \dfrac{4}{\sqrt[3]{8}} \\ \quad = \dfrac{4}{2} \\ f'(8) = 2 \text{ words}/\text{min} \end{array}}$

b. Interpret your answer.

Eight minutes after memorizing began, the total words memorized is increasing by 2 words per minute.