# Derivative Applications
## 3.2A – Marginal Average Cost, Revenue, & Profit

### The Bluth Company

#1) During the 1980s George Bluth's real estate development firm, The Bluth Company, was responsible for building homes for many influential people, including Saddam Hussein. During this decade The Bluth Company's revenue function was $R(x) = -x^2 + 5000x + 1{,}000{,}000$ dollars, where x is the number of homes sold.

a. Find *R(10)* and interpret your answer.
b. Find *AR(10)* and interpret your answer.
c. Find *MR(10)* and interpret your answer.
d. Find *MAR(10)* and interpret your answer.

a. $R(10) = -(10)^2 + 5000(10) + 1{,}000{,}000$

$= -100 + 50{,}000 + 1{,}000{,}000$

$R(10) = \$1{,}049{,}900$

When 10 homes have been sold, the total revenue of The Bluth Company is $1,049,900.

b. $AR(x) = \frac{R(x)}{x} = \frac{-x^2 + 5000x + 1{,}000{,}000}{x}$

$AR(x) = -x + 5000 + \frac{1{,}000{,}000}{x}$

$AR(10) = -(10) + 5000 + \frac{1{,}000{,}000}{(10)}$

$= 4990 + 100{,}000$

$AR(10) = \$104{,}990$

When 10 homes have been sold, the average revenue of The Bluth Company is $104,990 per home.

c. $R(x) = -x^2 + 5000x + 1{,}000{,}000$

$MR(x) = -2x + 5000$

$MR(10) = -2(10) + 5000$

$= -20 + 5000$

$MR(10) = \$4980$ per home

When 10 homes have been sold, The Bluth Company's total revenue is increasing by $4980 per home sold.

OR

When 10 homes have been sold, The Bluth Company's revenue for the next home is $4980.

d. $AR(x) = -x + 5000 + 1{,}000{,}000x^{-1}$

$MAR(x) = -1 - 1{,}000{,}000x^{-2}$

$MAR(x) = -1 - \frac{1{,}000{,}000}{x^2}$

$MAR(10) = -1 - \frac{1{,}000{,}000}{(10)^2}$

$= -1 - \frac{1{,}000{,}000}{100}$

$= -1 - 10{,}000$

$MAR(10) = -\$10{,}001$/home

When 10 homes have been sold, The Bluth Company's average revenue per home is decreasing by $10,001 per home.

### Michael Bluth's Whistles

#2) Tired of the deceit, lies and general misconduct of his employees, Michael Bluth decides to open a factory to make whistles. He hopes to empower everyone in The Bluth Company to be a figurative whistle blower by getting each of them to literally blow a whistle. Michael can produce whistles at a cost of $0.25 each with fixed costs $75 per week.

a. Find the cost function.
b. Find the average cost function.
c. Find the marginal average cost function.
d. Evaluate *MAC(x)* at $x = 500$ and interpret your answer.

x = # of whistles

a. $C(x) = 0.25x + 75$

b. $AC(x) = \frac{C(x)}{x} = \frac{0.25x + 75}{x}$

$AC(x) = 0.25 + \frac{75}{x}$

c. $AC(x) = 0.25 + 75x^{-1}$

$MAC(x) = -75x^{-2}$

$MAC(x) = \frac{-75}{x^2}$

d. $MAC(500) = \frac{-75}{(500)^2}$

$= \frac{-75}{250{,}000}$

$MAC(500) = -.\$0003$/whistle

When Michael Bluth's factory has produced 100 whistles, the average cost per whistle is decreasing by $0.0003 per whistle made.

# Derivative Applications
## 3.2A – Marginal Average Cost, Revenue, & Profit

**Bluth's Original Frozen Bananas**

#3) George Michael Bluth can produce original frozen bananas at a cost of $0.50 each with fixed costs $400 per week.

a. Find the cost function.
b. Find the average cost function.
c. Find the marginal average cost function.
d. Evaluate *MAC(x)* at $x = 20$ and interpret your answer.

$x$ = # of bananas

a.

$C(x) = 0.50x + 400$

b.

$AC(x) = \frac{C(x)}{x} = \frac{0.50x + 400}{x}$

$AC(x) = 0.50 + \frac{400}{x}$

c. $AC(x) = 0.50 + 400x^{-1}$

$MAC(x) = -400x^{-2}$

$MAC(x) = \frac{-400}{x^2}$

d. $MAC(20) = \frac{-400}{(20)^2}$

$= \frac{-400}{400}$

$MAC(20) = -\$1$ per banana

When George Michael has sold 20 Bluth's Original Frozen Bananas, the average cost of each frozen banana is decreasing by $1 per frozen banana made.

**Bob Loblaw**

#4) Bob Loblaw is an attorney for The Bluth Company. In order to gain more clients, he advertises on billboards throughout the state of California. He wants his slogan "You don't need double talk; you need Bob Loblaw" to be seen by every resident of the state. Bob Loblaw can produce billboards at a cost of $5000 each with fixed costs $10,000 per week.

a. Find the cost function.
b. Find the average cost function.
c. Find the marginal average cost function.
d. Evaluate *MAC(x)* at $x = 500$ and interpret your answer.

$x$ = # of billboards

a. $C(x) = 5000x + 10{,}000$

b. $AC(x) = \frac{C(x)}{x} = \frac{5000x + 10{,}000}{x}$

$AC(x) = 5000 + \frac{10{,}000}{x}$

c. $AC(x) = 5000 + 10{,}000x^{-1}$

$MAC(x) = -10{,}000x^{-2}$

$MAC(x) = \frac{-10{,}000}{x^2}$

d. $MAC(500) = \frac{-10{,}000}{(500)^2}$

$= \frac{-10{,}000}{250{,}000}$

$MAC(500) = \$0.04$ / billboard

When 500 Bob Loblaw billboards have been made, the average cost per billboard is decreasing by 4¢ per billboard made.

# Derivative Applications
## 3.2A – Marginal Average Cost, Revenue, & Profit

**GOB Illusions**

#5) George Oscar Bluth, or G.O.B. for short, sells the secret to his illusions on the playgrounds of neighboring elementary schools, which is no doubt the reason he's been kicked out of the The Alliance of Magicians. The revenue from his illustrious illusions is given by the function $R(x) = -x^2 + 30x + 3$ dollars, where $x$ is the number of dimwitted children foolish enough to buy his tricks, er, I mean illusions.

a. Find the average revenue function.
b. Find the marginal average revenue function.
c. Evaluate *MAR(5)* and interpret your answer.

x = # of dimwits

a. $AR(x) = \frac{R(x)}{x} = \frac{-x^2+30x+3}{x}$

$AR(x) = -x + 30 + \frac{3}{x}$

b. $AR(x) = -x + 30 + 3x^{-1}$

$MAR(x) = -1 - 3x^{-2}$

$MAR(x) = -1 - \frac{3}{x^2}$

c. $MAR(5) = -1 - \frac{3}{(5)^2}$

$= -1 - \frac{3}{25}$

$MAR(5) = -\$1.12$ / dimwit

When G.O.B has sold 5 illustrious illusions, his average revenue is decreasing by $1.12 per dimwit dumb enough to buy his trick (I mean illusion).

**Buster's Claw Crane Toys**

#6) Buster Bluth became obsessed with winning toys from playing the claw crane machine. He became so good at the claw crane that he found himself swimming in fluffy toys. So, to help the Bluth family, he decided to generate some revenue. The revenue, *R(x)* measured in US currency, generated from selling *x* fluffy toys is *R(x) = 5x + 4*.

a. Find the average revenue function.
b. Find the marginal average revenue function.
c. Evaluate *MAR(15)* and interpret your answer.

a. $AR(x) = \frac{R(x)}{x} = \frac{5x+4}{x}$

$AR(x) = 5 + \frac{4}{x}$

b. $AR(x) = 5 + 4x^{-1}$

$MAR(x) = -4x^{-2}$

$MAR(x) = \frac{-4}{x^2}$

c. $MAR(15) = \frac{-4}{(15)^2}$

$= \frac{-4}{225}$

$MAR(15) \simeq -\$.018$ / fluffy toy

When Buster has sold 15 fluffy toys, his average revenue per toy is decreasing by about 1.8¢ per fluffy toy sold.

# Derivative Applications
## 3.2A – Marginal Average Cost, Revenue, & Profit

**Tobias Fünke - Actor Extraordinaire**

#7) Thespian and former doctor of psychology, Tobias Funke, is a man of many talents. While studying under penny pincher Carl Weathers, Tobias was able to start of business selling rancid, runny garbage as vegetable soup to homeless people. His profit function is $P(x) = 0.20x - 5$ dollars, where $x$ is the gallons of "soup" sold.

a. Find *P(100)* and interpret your answer.
b. Find *AP(100)* and interpret your answer.
c. Find *MP(100)* and interpret your answer.
d. Find *MAP(100)* and interpret your anwer.

a. $P(100) = 0.20(100) - 5$
$= 20 - 5$
$P(100) = \$15$

When Tobias has sold 100 gallons of soup, his total profit is $15.

b. $AP(x) = \frac{P(x)}{x}$
$= \frac{0.20x - 5}{x}$
$AP(x) = 0.20 - \frac{5}{x}$

$AP(100) = 0.20 - \frac{5}{100}$
$= 0.20 - 0.05$
$AP(100) = \$0.15$

When Tobias has sold 100 gallons of soup, his average profit is 15¢ per gallon of soup.

c. $MP(100) = \$0.20$/gallon of soup

When Tobias has sold 100 gallons of soup, his total profit is increasing by 20¢ per gallon of soup sold.

OR

When Tobias has sold 100 gallons of soup, his profit on the next gallon of soup is 20¢.

d. $AP(x) = 0.20 - 5x^{-1}$
$MAP(x) = 5x^{-2}$
$MAP(x) = \frac{5}{x^2}$

$MAP(100) = \frac{5}{(100)^2}$
$= \frac{5}{10000}$
$MAP(100) = \$.0005$/gallon

When Tobias has sold 100 gallons of soup, his average profit is increasing by 0.05¢ per gallon of soup sold.

**Maeby Fünke's BS**

#8) Maeby has an alter ego where she pretends to be a wheelchair bound girl suffering from a disease called BS. She uses this con to "earn" money through donations. Her profit *P(x)*, measured in US currency, is $P(x) = 20x - 2500$ where $x$ is the number of days she pretends to have BS.

a. Find the average profit function.
b. Find the marginal average profit function.
c. Evaluate *MAP(200)* and interpret your answer.

a. $AP(x) = \frac{P(x)}{x}$
$= \frac{20x - 2500}{x}$
$AP(x) = 20 - \frac{2500}{x}$

b. $AP(x) = 20 - 2500x^{-1}$
$MAP(x) = 2500x^{-2}$
$MAP(x) = \frac{2500}{x^2}$

c. $MAP(200) = \frac{2500}{(200)^2}$
$= \frac{2500}{40000}$
$MAP(200) = \$.0625$/day

On day 200 days of pretending to have BS, Maeby's average profit is increasing by 6.25¢ per day.