Derivative Applications
3.2 – Marginal Average Cost, Revenue, & Profit

Average Cost, Average Revenue & Average Profit
These are often useful to calculate the average cost per unit, the average revenue per unit, and the average profit per unit, denoted by AC(x), AR(x), and AP(x).

Average Cost

$$AC(x) = \frac{C(x)}{x}$$

Ex: If AC(45 sneakers) = $30.

When the 45th pair of sneakers has been produced, the average cost is $30 per pair of sneakers.

Average Revenue

$$AR(x) = \frac{R(x)}{x}$$

Ex: If AR(45 sneakers) = $100.

When the 45th pair of sneakers has been produced, the average revenue is $100 per pair of sneakers.

Average Profit

$$AP(x) = \frac{P(x)}{x}$$

Ex: If AP(45 sneakers) = $70.

When the 45th pair of sneakers has been produced, the average profit is $70 per pair of sneakers.

Marginal Average Cost
The marginal average cost reveals how much the average cost of producing an item is changing at any given moment.

$$MAC(x) = \left(\frac{C(x)}{x}\right)'$$

Ex: If MAC(45 sneakers) = -$3.

When the 45th pair of sneakers has been produced, the average cost is decreasing by $3 per pair of sneakers.

Marginal Average Revenue
The marginal average revenue reveals how much the average revenue from producing an item is changing at any given moment.

$$MAR(x) = \left(\frac{R(x)}{x}\right)'$$

Ex: If MAR(45 sneakers) = $2

When the 45th pair of sneakers has been produced, the average revenue is increasing by $2 per pair of sneakers.

Marginal Average Profit
The marginal average profit reveals how much the average profit from producing an item is changing at any given moment.

$$MAP(x) = \left(\frac{P(x)}{x}\right)'$$

Ex: If MAP(45 sneakers) = $5

When the 45th pair of sneakers has been produced, the average profit is increasing by $5 per pair of sneakers.

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Shirt Company

#1) It costs a shirt company $2 to produce each shirt, and fixed costs are $10,000.

a. Find C(100) and interpret your answer.

x = # of shirts

$C(x) = 2x + \$10{,}000$

$C(100) = 2(100) + 10{,}000$

$= 200 + 10{,}000$

$C(100) = \$10{,}200$

When 100 shirts have been made, the total cost is $10,200.

b. Find MC(100) and interpret your answer.

$MC(x) = 2$

$MC(100) = \$2/\text{shirt}$

When 100 shirts have been made, the total cost is increasing by $2 per shirt.

or

When 100 shirts have been made, the cost to make the next shirt is $2.

c. Find the average cost at x = 100 and interpret your answer.

$AC(x) = \frac{C(x)}{x}$

$AC(x) = \frac{2x + 10{,}000}{x}$

$AC(100) = \frac{2(100) + 10{,}000}{100}$

$= \frac{200 + 10{,}000}{100}$

$= \frac{10200}{100}$

$AC(100) = 102$

When 100 shirts have been made, the average cost to make each shirt is $102.

d. Find the marginal average cost at x = 100 and interpret your answer.

$MAC(x) = -10{,}000x^{-2}$

$MAC(x) = \frac{-10{,}000}{x^2}$

$MAC(100) = \frac{-10{,}000}{(100)^2}$

$= \frac{-10{,}000}{10{,}000}$

$MAC(100) = -\$1$ per shirt

When 100 shirts have been produced, the average cost per shirt is decreasing by $1 per shirt.

McSlapping

#2) Mr McConnell sells head slaps which generates revenue shown by the function $R(x) = -x^2 + 600x + 800$ dollars where x is the number of slaps sold/given.

a. Find R(45) and interpret your answer.

$R(45) = -(45)^2 + 600(45) + 800$

$= -2025 + 27000 + 800$

$R(45) = \$25{,}775$

When 45 head slaps have been sold, the total revenue is $25,775.

b. Find MR(45) and interpret your answer.

$MR(x) = -2x + 600$

$MR(45) = -2(45) + 600$

$= -90 + 600$

$MR(45) = \$510/\text{slap}$

When 45 head slaps have been sold, the total revenue is increasing by $510 per slap.

or

When 45 head slaps have been sold, the revenue from the next slap is $510.

c. Find the average revenue at x = 45 and interpret your answer.

$AR(x) = \frac{R(x)}{x} = \frac{-x^2 + 600x + 800}{x}$

$AR(x) = -x + 600 + \frac{800}{x}$

$AR(45) = -(45) + 600 + \frac{800}{(45)}$

$= 555 + \frac{800}{45}$

$AR(45) \approx \$572.78$

When 45 head slaps have been sold, the average revenue per slap is $572.78.

d. Find the marginal average revenue at x = 45 and interpret your answer.

$AR(x) = -x + 600 + 800x^{-1}$

$MAR(x) = -1 - 800x^{-2}$

$MAR(x) = -1 - \frac{800}{x^2}$

$MAR(45) = -1 - \frac{800}{(45)^2}$

$= -1 - \frac{800}{2025}$

$MAR(45) = -\$1.40/\text{slap}$

When 45 slaps have been sold, the average revenue per slap is decreasing by $1.40 per slap.