Derivative Applications
3.3A – Higher Order Derivatives

A: Find the first four derivatives of each function.

#1) $f(x) = x^4 + 3x - 8$ (Use Leibniz)

$$\frac{df}{dx} = 4x^3 + 3$$

$$\frac{d^2f}{dx^2} = 12x^2$$

$$\frac{d^3f}{dx^3} = 24x$$

$$\frac{d^4f}{dx^4} = 24$$

#2) $y = \frac{1}{60}x^5 - \frac{1}{24}x^4 + x$ (Use Leibniz)

$$\frac{dy}{dx} = \frac{5}{60}x^4 - \frac{4}{24}x^3 + 1$$

$$= \frac{1}{12}x^4 - \frac{1}{6}x^3 + 1$$

$$\frac{d^2y}{dx^2} = \frac{1}{3}x^3 - \frac{1}{2}x^2$$

$$\frac{d^3y}{dx^3} = x^2 - x$$

$$\frac{d^4y}{dx^4} = 2x - 1$$

#3) $f(x) = \sqrt{x^6}$ (Use Newton)

$$f(x) = x^{6/2} = x^3$$

$$f'(x) = 3x^2$$

$$f''(x) = 6x$$

$$f'''(x) = 6$$

$$f^{(4)}(x) = 0$$

B: Find $f''(2)$.

#4) $f(x) = \frac{x+1}{x}$

$$f'(x) = \frac{(x+1)' \cdot x - (x+1)x'}{x^2}$$

$$= \frac{(1)x - (x+1)(1)}{x^2}$$

$$= \frac{x - x - 1}{x^2}$$

$$f'(x) = \frac{-1}{x^2}$$

$$f'(x) = -x^{-2}$$

$$f''(x) = 2x^{-3}$$

$$f''(x) = \frac{2}{x^3}$$

$$f''(2) = \frac{2}{(2)^3}$$

$$= \frac{2}{8}$$

$$f''(2) = \frac{1}{4}$$

#5) $f(x) = \frac{x-9}{3x}$

$$\boxed{f'(x) = \frac{(x-9)'(3x) - (x-9)(3x)'}{(3x)^2} = \frac{(1)(3x) - (x-9)(3)}{9x^2} = \frac{3x - 3x + 27}{9x^2} = \frac{27}{9x^2}, \quad f'(x) = \frac{3}{x^2}}$$

$$f'(x) = 3x^{-2}$$

$$\boxed{f''(x) = -6x^{-3}, \quad f''(x) = \frac{-6}{x^3}}$$

$$\boxed{f''(2) = \frac{-6}{(2)^3} = \frac{-6}{8}, \quad f''(2) = -\frac{3}{4}}$$

#6) $f(x) = \frac{7}{x^2}$

$$f(x) = 7x^{-2}$$

$$\boxed{f'(x) = -14x^{-3}}$$

$$f''(x) = 42x^{-4}$$

$$f''(x) = \frac{42}{x^4}$$

$$f''(2) = \frac{42}{(2)^4}$$

$$= \frac{42}{16}$$

$$f''(2) = \frac{21}{8}$$

Derivative Applications
3.3A – Higher Order Derivatives

C: Find the first and second derivative.

#7) $f(x) = (x^2 - 1)(x^2 + 1)$

$f(x) = x^4 - 1$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

#8) $f(x) = \frac{64}{\sqrt{x}}$

$f(x) = 64x^{-1/2}$

$f'(x) = -32x^{-3/2}$

$f'(x) = \frac{-32}{\sqrt{x^3}}$

$f''(x) = 48x^{-5/2}$

$f''(x) = \frac{48}{\sqrt{x^5}}$

#9) $f(x) = \frac{x}{x-1}$

$f'(x) = \frac{(x)'(x-1) - x(x-1)'}{(x-1)^2}$

$= \frac{(1)(x-1) - x(1)}{(x-1)^2}$

$= \frac{x-1-x}{(x-1)^2}$

$f'(x) = \frac{-1}{(x-1)^2}$

$f''(x) = \frac{(-1)'(x-1)^2 - (-1)\left[(x-1)^2\right]'}{\left[(x-1)^2\right]^2}$

$= \frac{0(x-1)^2 + 1(x^2-2x+1)'}{(x-1)^4}$

$= \frac{2x-2}{(x-1)^4}$

$= \frac{2(x-1)}{(x-1)^4}$

$f''(x) = \frac{2}{(x-1)^3}$

D: Evaluate.

#10) $\frac{d^2}{dr^2}(\pi r^2)|_{r=2}$

$\frac{d}{dr}(\pi r^2) = 2\pi r$

$\frac{d^2}{dr^2}(\pi r^2) = 2\pi$

$\frac{d^2}{dr^2}(\pi r^2)\Big|_{r=2} = 2\pi$

#11) $\frac{d^2}{dr^2}(r^5)|_{r=2}$

$\frac{d}{dr}(r^5) = 5r^4$

$\frac{d^2}{dr^2}(r^5) = 20r^3$

$\frac{d^2}{dr^2}(r^5)\Big|_{r=2} = 20(2)^3$

$= 20(8)$

$\frac{d^2}{dr^2}(r^5)\Big|_{r=2} = 160$

Creepers in Duckland

#12) George decides to play Minecraft after a long dormant period of Minecraft agnosticism. Upon entering Duckland, his personal Minecraft world, he was shocked to see that hackers had invaded his game. The Hip-Hop-Anonymous hacker clan had populated his game with Creepers. The number of Creepers in Duckland is $p(t) = 99t^{4/3}$ thousand, where t is the number of days after the clan started their assault.

a. Find $p(8)$ and interpret your answer.
b. Find $p'(8)$ and interpret your answer.
c. Find $p''(8)$ and interpret your answer.

$p(t) = \#$ of creepers
$t = \#$ of days

a. $p(8) = 99(\sqrt[3]{(8)})^4$
$= 99(2)^4$
$= 99(16)$
$p(8) = 1584$ thousand

Eight days after the Hip-Hop-Anonymous clan attacked, there were a total of 1,584,000 Creepers in Duckland.

b. $p'(t) = \frac{4}{\cancel{3}}(\overset{33}{\cancel{99}})t^{1/3}$
$p'(t) = 132t^{1/3}$
$p'(8) = 132\sqrt[3]{(8)}$
$= 132(2)$
$p'(8) = 264$ thousand creepers/day

Eight days after a hacker attack, the total number of Creepers in Duckland is increasing by 264,000 creepers per day.

c. $p''(t) = \frac{132}{3}t^{-2/3}$
$p''(t) = \frac{132}{3(\sqrt[3]{t})^2}$

$p''(8) = \frac{132}{3(\sqrt[3]{8})^2}$
$= \frac{132}{3(2)^2}$
$= \frac{\cancel{132}\cdot 33}{3\cdot\cancel{4}_1}$
$= \frac{33}{3}$
$p''(8) = 11$ thousand creepers/day/day

Eight days after the hackers invaded Duckland, the Creeper growth rate is increasing by 11,000 Creepers per day each day.

George's Fever

#13) After seeing his beloved Duckland torn to bits by Creepers, George's immune system became weak as he developed a fever. George's temperature t hours after discovering the demise of Duckland is $T(t) = \frac{10}{\sqrt{t}} + 98$ degrees F.

a. Find $T(1)$ and interpret your answer.
b. Find $T'(1)$ and interpret your answer.
c. Find $T''(1)$ and interpret your answer.

$T(t) = {}^\circ F$
$t =$ hours

a. $T(1) = \frac{10}{\sqrt{1}} + 98$
$= 10 + 98$
$T(1) = 108^\circ F$

One hour after discovering Creepers in Duckland, George's temperature is 108° F.

b. $T(t) = 10t^{-\frac{1}{2}} + 98$
$T'(t) = -5t^{-3/2}$

$T'(t) = \frac{-5}{\sqrt{t^3}}$
$T(1) = \frac{-5}{\sqrt{(1)^3}}$
$= \frac{-5}{1}$
$T(1) = -5^\circ F/\text{hour}$

One hour after Creepers invaded Duckland, George temperature is decreasing by 5°F per hour.

c. $T''(t) = \frac{15}{2}t^{-5/2}$
$= \frac{15}{2\sqrt{t^5}}$

$T''(1) = \frac{15}{2\sqrt{(1)^5}}$
$= \frac{15}{2}$
$= 7.5^\circ F/\text{hour}/\text{hour}$

One hour after Creepers invaded Duckland, the rate at which George's temperature is changing is increasing by 7.5°F per hour each hour.

Bacteria Paradise

#14) While in a weak, decrepit state, George lay in bed for days morning the loss of his lovely Duckland. Depressed to the point of losing all motivation to take care of his personal hygiene, George began to stink. Bacteria started to have a rave party on George. The population of the bacteria is predicted to be $p(t) = 4t^{3/2} + 5$ critters, where t is the number of hours after George lost all motivation and self-respect.

a. Find $p(2)$ and interpret your answer.
b. Find $p'(2)$ and interpret your answer.
c. Find $p''(2)$ and interpret your answer.

a. $p(2) = 4(\sqrt{2})^3 + 5$

$p(2) \approx 16.3$ critters

Two hours after George lost all respect for himself, the total number of critters on him is about 16.3.

b. $p'(t) = 6t^{\frac{1}{2}}$

$p'(2) = 6\sqrt{2}$

$p'(2) \approx 8.5$ critters/hour

Two hours after George lost all respect for himself, the total number of critters on him is increasing by about 8.5 critters per hour.

c. $p''(t) = 3t^{-\frac{1}{2}}$

$p''(2) = \frac{3}{\sqrt{2}}$

$p''(2) \approx 2.1$ critters/hour/hour

Two hours after George lost all respect for himself, the growth rate of the critters is increasing by 2.1 critters per hour each hour.

Soup Du Jour

#15) While smelling the aroma of George's funk seeping out of his room, his momma became concerned for his wellbeing. She decided to make a delicious batch of soup to cheer him up. After taking a sip of the soup, George smiled for the first time in days. "What kind of soup is this momma?" he asked. "Oh, it's duck soup dear." Hearing the words "duck soup" caused George to immediately spit out the foul soup. George became outraged at the insensitivity his mother showed toward Duckland by serving him soup ala duck. The duck soup's temperature t minutes after being spat out of his mouth is $T(t) = \frac{-10}{\sqrt{t}} + 120$ degrees F.

a. Find $T(4)$ and interpret your answer.
b. Find $T'(4)$ and interpret your answer.
c. Find $T''(4)$ and interpret your answer.

a. $T(4) = \frac{-10}{\sqrt{4}} + 120$

$= \frac{-10}{2} + 120$

$= -5 + 120$

$T(4) = 115°$

Four minutes after the duck soup was spat out of George's mouth, the soup was 115°F.

b. $T(t) = -10t^{-\frac{1}{2}} + 120$

$T'(t) = 5t^{-3/2}$

$T'(4) = \frac{5}{(\sqrt{4})^3}$

$= \frac{5}{(2)^3}$

$= \frac{5}{8}$

$T'(4) = .625$ °F/minute

c. $T''(t) = \frac{-15}{2}t^{-5/2}$

$T''(t) = \frac{-15}{2(\sqrt{t})^5}$

$T''(4) = \frac{-15}{2(\sqrt{4})^5}$

$= \frac{-15}{2(2)^5}$

$= \frac{-15}{2(32)}$

$= \frac{-15}{64}$

$T''(4) \approx -.23$

Four minutes after the duck soup was spat out of George's mouth, the rate of change of the soup's temperature is decreasing by was 0.23°F per minute.