Derivative Applications
3.3 – Higher Order Derivatives

Newton Notation

1st Derivative	f' or y'
2nd Derivative	f'' or y''
3rd Derivative	f''' or y'''
4th Derivative	$f^{(4)}$ or $y^{(4)}$
Nth Derivative	$f^{(n)}$ or $y^{(n)}$

Leibniz Notation

1st Derivative	$\frac{dy}{dx}$ or $\frac{d}{dx}f$
2nd Derivative	$\frac{d^2y}{dx^2}$ or $\frac{d^2}{dx^2}f$
3rd Derivative	$\frac{d^3y}{dx^3}$ or $\frac{d^3}{dx^3}f$
4th Derivative	$\frac{d^4y}{dx^4}$ or $\frac{d^4}{dx^4}f$
Nth Derivative	$\frac{d^ny}{dx^n}$ or $\frac{d^n}{dx^n}f$

Ex A: Find the first four derivatives of each function.

#1) $f(x) = x^4 - x^3 + 6x^2 - x + 1$ (Use Newton)

$f'(x) = 4x^3 - 3x^2 + 12x - 1$

$f''(x) = 12x^2 - 6x + 12$

$f'''(x) = 24x - 6$

$f^{(4)}(x) = 24$

#2) $y = 3x^{-1/2}$ (Use Newton)

$y' = -\frac{3}{2}x^{-3/2}$

$y'' = \frac{9}{4}x^{-5/2}$

$y''' = -\frac{45}{8}x^{-7/2}$

$y^{(4)} = \frac{315}{16}x^{-9/2}$

#3) $f(x) = x^3 - 2x^2 + 6x - 11$ (Use Leibniz)

$\frac{df}{dx} = 3x^2 - 4x + 6$

$\frac{d^2f}{dx^2} = 6x - 4$

$\frac{d^3f}{dx^3} = 6$

$\frac{d^4f}{dx^4} = 0$

#4) $y = \frac{1}{x}$ (Use Leibniz)

$y = x^{-1}$

$\frac{dy}{dx} = -1 \cdot x^{-2} = -\frac{1}{x^2}$

$\frac{d^2y}{dx^2} = -2 \cdot x^{-3} = -\frac{2}{x^3}$

$\frac{d^3y}{dx^3} = 6x^{-4} = \frac{6}{x^4}$

$\frac{d^4y}{dx^4} = -24x^{-5} = -\frac{24}{x^5}$

#5) $\frac{d^2}{dx^2}(x^3 + x^2 + x - 1)|_{x=1}$

$\frac{d}{dx}(x^3 + x^2 + x - 1) = 3x^2 + 2x + 1$

$\frac{d^2}{dx^2}(x^3 + x^2 + x - 1) = 6x + 2$

$\frac{d^2}{dx^2}(x^3 + x^2 + x - 1)\Big|_{x=1} = 6(1) + 2$

$= 6 + 2$

$\frac{d^2}{dx^2}(x^3 + x^2 + x - 1)\Big|_{x=1} = 8$

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G Pop

The population of Gnadenhutten boomed after the arrival of the Gnaden Family Store. The population of the village is predicted to be $p(x) = \frac{x^2-1}{10x+10}$ thousand people, where *t* is the number of years after the Gnaden Family Store moved to town.

a. Find $p(8)$ and interpret your answer.

b. Find $p'(8)$ and interpret your answer.

c. Find $p''(8)$ and interpret your answer.

Chocolate Fever

The temperature of a chocolate bar is $y = \frac{x}{x+1}$ hundred degrees *F*, where *x* is the seconds after the chocolate was taken out of the freezer and put under someone's armpit.

a. Find $T(1)$ and interpret your answer.

b. Find $T'(1)$ and interpret your answer.

c. Find $T''(1)$ and interpret your answer.