Derivative Applications
3.3 – Higher Order Derivatives

Newton Notation

1st Derivative	f' or y'
2nd Derivative	f'' or y''
3rd Derivative	f''' or y'''
4th Derivative	$f^{(4)}$ or $y^{(4)}$
Nth Derivative	$f^{(n)}$ or $y^{(n)}$

Leibniz Notation

1st Derivative	$\frac{dy}{dx}$ or $\frac{d}{dx}f$
2nd Derivative	$\frac{d^2y}{dx^2}$ or $\frac{d^2}{dx^2}f$
3rd Derivative	$\frac{d^3y}{dx^3}$ or $\frac{d^3}{dx^3}f$
4th Derivative	$\frac{d^4y}{dx^4}$ or $\frac{d^4}{dx^4}f$
Nth Derivative	$\frac{d^ny}{dx^n}$ or $\frac{d^n}{dx^n}f$

Ex A: Find the first four derivatives of each function.

#1) $f(x) = x^4 - x^3 + 6x^2 - x + 1$ (Use Newton)

$f'(x) = 4x^3 - 3x^2 + 12x - 1$

$f''(x) = 12x^2 - 6x + 12$

$f'''(x) = 24x - 6$

$f^{(4)}(x) = 24$

#2) $y = 3x^{-1/2}$ (Use Newton)

$y' = -\frac{3}{2}x^{-3/2}$

$y'' = \frac{9}{4}x^{-5/2}$

$y''' = \frac{-45}{8}x^{-7/2}$

$y^{(4)} = \frac{315}{16}x^{-9/2}$

#3) $f(x) = x^3 - 2x^2 + 6x - 11$ (Use Leibniz)

$\frac{df}{dx} = 3x^2 - 4x + 6$

$\frac{d^2f}{dx^2} = 6x - 4$

$\frac{d^3f}{dx^3} = 6$

$\frac{d^4f}{dx^4} = 0$

#4) $y = \frac{1}{x}$ (Use Leibniz)

$y = x^{-1}$

$\frac{dy}{dx} = -1 \cdot x^{-2} = -\frac{1}{x^2}$

$\frac{d^2y}{dx^2} = -2 \cdot x^{-3} = -\frac{2}{x^3}$

$\frac{d^3y}{dx^3} = 6x^{-4} = \frac{6}{x^4}$

$\frac{d^4y}{dx^4} = -24x^{-5} = \frac{-24}{x^5}$

#5) $\frac{d^2}{dx^2}(x^3 + x^2 + x - 1)|_{x=1}$

$\frac{d}{dx}(x^3+x^2+x-1) = 3x^2+2x+1$

$\frac{d^2}{dx^2}(x^3+x^2+x-1) = 6x+2$

$\frac{d^2}{dx^2}(x^3+x^2+x-1)\Big|_{x=1} = 6(1)+2$

$= 6+2$

$\frac{d^2}{dx^2}(x^3+x^2+x-1)\Big|_{x=1} = 8$

G Pop

The population of Gnadenhutten boomed after the arrival of the Gnaden Family Store. The population of the village is predicted to be $p(t) = \frac{t^2-1}{10t+10}$ thousand people, where *t* is the number of years after the Gnaden Family Store moved to town.

t = years since GFS moved to town
p = people in thousands

a. Find $p(8)$ and interpret your answer.

$$p(t) = \frac{\cancel{(t+1)}(t-1)}{10\cancel{(t+1)}} = \frac{t-1}{10}$$

$$p(8) = \frac{(8)-1}{10} = \frac{7}{10} = .7 \text{ thousand people}$$

Eight years after GFS moved to town the population is 700 people

b. Find $p'(8)$ and interpret your answer.

$$p(t) = \frac{1}{10}t - \frac{1}{10}$$

$$p'(t) = \frac{1}{10} = .1 \text{ thousand people/year}$$

Eight years after GFS moved to town the population is increasing by 100 people per year.

c. Find $p''(8)$ and interpret your answer.

$$p''(t) = 0$$

$$p''(8) = 0 \text{ thousand people/year/year}$$

Eight years after GFS moved to town the population growth rate is constant.

Chocolate Fever

The temperature of a chocolate bar is $T(x) = \frac{x}{x+1}$ hundred degrees *F*, where *x* is the seconds after the chocolate was taken out of the freezer and put under someone's armpit.

x = seconds after freezer (INTO PIT)
T = °F in hundreds

a. Find $T(1)$ and interpret your answer.

$$T(1) = \frac{1}{(1)+1} = \frac{1}{2} = .5 \text{ hundred °F}$$

One second after putting the frozen chocolate under a pit, its temperature is 50° F.

b. Find $T'(1)$ and interpret your answer.

1

$$T'(x) = \frac{(x)'(x+1) - x(x+1)'}{(x+1)^2}$$

$$= \frac{1(x+1) - x(1)}{(x+1)^2}$$

$$= \frac{x+1-x}{(x+1)^2}$$

$$T'(x) = \frac{1}{(x+1)^2}$$

2

$$T'(1) = \frac{1}{[(1)+1]^2} = \frac{1}{[2]^2} = \frac{1}{4}$$

$$T'(1) = .25 \frac{\text{hundred °F}}{\text{second}}$$

One second after putting the frozen chocolate under a pit, its temperature is increasing by 25°F per second.

c. Find $T''(1)$ and interpret your answer.

1

$$T''(x) = \frac{(1)'(x^2+2x+1) - 1(x^2+2x+1)'}{[(x+1)^2]^2}$$

$$= \frac{0(x^2+2x+1) - (2x+2)}{(x+1)^4}$$

$$= \frac{-(2x+2)}{(x+1)^4}$$

$$= \frac{-2(x+1)}{(x+1)^4}$$

$$T''(x) = \frac{-2}{(x+1)^3}$$

2

$$T''(1) = \frac{-2}{[(1)+1]^3} = \frac{-2}{[2]^3} = \frac{-2}{8}$$

$$T''(1) = -.25 \text{ °F/sec}^2$$

One second after putting the frozen chocolate under a pit, its temperature change is decreasing by 25°F per second each second.