

# Derivative Applications

## 3.4A – Distance, Velocity, Acceleration & Other Stuff

### Thwomp Ruins

#1) Entering the final race of the Mushroom Cup, Luigi is sitting alone at first place. After driving on Thwomp Ruins for  $t$  seconds Luigi is  $s(t) = 15t^2 - 2t^3$  meters due east of the starting point of the race. (for  $0 < t < 9$ ).

- Find Luigi's velocity at 2 seconds.
- Find his velocity at 8 seconds.
- Find his acceleration at 8 seconds and interpret your answer as it relates to velocity.

$s(t) = \text{meters}$   
 $t = \text{seconds}$

a.

$$v(t) = 30t - 6t^2$$

$$v(2) = 30(2) - 6(2)^2$$

$$= 60 - 6(4)$$

$$= 60 - 24$$

$$v(2) = 36 \text{ m/s east}$$

Two seconds after the race began, Luigi's velocity is 36 meters per second heading east.

b.

$$v(8) = 30(8) - 6(8)^2$$

$$= 240 - 6(64)$$

$$= 240 - 384$$

$$= -144 \text{ m/s east}$$

$$v(8) = 144 \text{ m/s west}$$

Eight seconds after the race began, Luigi's velocity is 144 meters per second heading west.

c.

$$a(t) = 30 - 12t$$

$$a(8) = 30 - 12(8)$$

$$= 30 - 96$$

$$a(8) = -66 \text{ m/s}^2$$

Eight seconds after the race began, Luigi's velocity is decreasing by 66 meters per second each second.

### Vomit

#2) After the dreaded turn eleven, Luigi spins his kart in multiple 360s due to a well-placed Donkey Kong banana peel. Sick to his stomach, Luigi ralphs.

Vomit is upchucked vertically in the air to a height of  $s(t) = 64t - 16t^2$  feet at  $t$  seconds.

- Find the vomit's distance at 1 second.
- Find the vomit's velocity at 1 second
- Find the vomit's acceleration at 1 second and interpret your answer as it relates to velocity.

$s(t) = \text{feet}$   
 $t = \text{seconds}$

a.

$$s(1) = 64(1) - 16(1)^2$$

$$= 64 - 16(1)$$

$$= 64 - 16$$

$$s(1) = 48 \text{ feet}$$

One second after Luigi ralphed, the vomit has traveled a distance of 48 feet.

b.

$$v(t) = 64 - 32t$$

$$v(1) = 64 - 32(1)$$

$$= 64 - 32$$

$$v(1) = 32 \text{ ft/sec}$$

One second after Luigi blew chunks, the vomit's velocity is 32 feet per second and is going up.

c.

$$a(t) = -32$$

$$a(1) = -32 \text{ ft/sec}^2$$

One second after Luigi upchucked, the vomit's velocity is decreasing by 32 feet per second every second.

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### Overheated

#3) Trying to recover from his mishap on the dreaded turn eleven, Luigi tries to make up lost time by shoving every mushroom he can find into his turbo shoot. Unfortunately for Luigi, he over did it. The engine block began to overheat. He had no choice but to pull his kart off the track and park beside the giant piranha plants. The temperature of Luigi's kart  $t$  seconds after pulling over is  $H(t) = 484 - t^2$  degrees Fahrenheit.

$H(t) = \text{°F}$   
 $t = \text{seconds}$

- Find  $H(12)$  and interpret your answer.
- Find  $H'(12)$  and interpret your answer.
- Find  $H''(12)$  and interpret your answer.

a.

$$H(12) = 484 - (12)^2$$

$$= 484 - 144$$

$$H(12) = 340^\circ\text{F}$$

Twelve seconds after overheating his engine, the engine's temperature is  $340^\circ\text{F}$

b.  $H'(t) = -2t$

$$H'(12) = -2(12)$$

$$H'(12) = -24^\circ\text{F}/\text{min}$$

Twelve seconds after Luigi's engine overheated, the engine's temperature is decreasing by about  $24^\circ\text{F}$  per seconds.

c.  $H''(t) = -2^\circ\text{F}/\text{sec}^2$

$$H''(12) = -2^\circ\text{F}/\text{sec}^2$$

Twelve seconds after Luigi's engine block overheated, the rate of change of the engine's temperature is decreasing by  $2^\circ\text{F}$  per second every second.

### Saliva With a Twist

#4) While waiting for his kart to fully cool down, Luigi starts to feel a drip on his head. Looking up, he realizes a giant piranha plant is salivating on him. Too exhausted to get out of his kart, Luigi just sits there and takes it. Sadly, this particular giant piranha plant's saliva is ripe with lice. The population of lice chillin' on top of Luigi's head  $t$  seconds from the first drop of saliva is predicted to be  $L(t) = 3t^2 + 18t^{2/3}$

- Find  $L(5)$  and interpret your answer.  $L(t) = \text{lice}$
- Find  $L'(5)$  and interpret your answer.  $t = \text{seconds}$
- Find  $L''(5)$  and interpret your answer.

a.  $L(5) = 3(5)^2 + 18(5)^{2/3}$

$$= 3(25) + 18(5)^{2/3}$$

$$= 75 + 18(5)^{2/3}$$

$$L(5) \approx 127.6 \text{ lice}$$

Five seconds after the first saliva drop, there are nearly 127.6 lice on Luigi's head.

b.  $L'(t) = 6t + 12t^{-1/3}$

$$L'(5) = 6(5) + 12(5)^{-1/3}$$

$$= 30 + 12(5)^{-1/3}$$

$$L'(5) \approx 37 \text{ Lice/second}$$

Five seconds after the first saliva drop, the number of lice on Luigi's head is increasing by about 37 lice per second.

c.  $L''(t) = 6 - 4t^{-4/3}$

$$L''(5) = 6 - 4(5)^{-4/3}$$

$$L''(5) = 5.5 \text{ lice}/\text{sec}^2$$

Five seconds after the first saliva drop, the lice growth rate is increasing by about 5.5 lice per second every second.

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### European Swallow

#5) Luigi finally gets his kart and attitude cooled and reenters the race. But, today is not his day. Flying high above his kart is a European swallow carrying a bomb. That's right. A bomb. (The European swallow is actually a remote controlled cyborg controlled by Wario.) At Wario's command, the European swallow drops the bomb on Luigi. The height of the bomb at  $t$  seconds is  $s(t) = 202 - 16t^2$  feet (neglecting air resistance, of course).

- If Luigi's head is 10 feet above the ground, how long will it take the bomb to reach his noggin?
- What will the velocity of the bomb be when it impacts Luigi's skull?

$s(t) = \text{feet}$   
 $t = \text{seconds}$

a.

$$10 = 202 - 16t^2$$

$$-192 = -16t^2$$

$$12 = t^2$$

$$\pm\sqrt{12} = t$$

$$\pm 3.5 \text{ seconds} \approx t \quad (-3.5 \text{ makes no sense})$$

It will take about 3.5 seconds for the bomb to hit Luigi's head.

b.

$$v(t) = -32t$$

$$v(3.5) = -32(3.5)$$

$$v(3.5) = -112 \text{ ft/sec}$$

The impact velocity of the bomb will be 112 feet per second. (And obviously the bomb was going down)

### African Swallows

#6) Moments after Luigi extinguishes the flames on his kart, he is back on track (pun) to continue his quest for a racing trophy. Flying above Thwomp Ruins is two, yes two, migratory African swallows. These two particular migratory African swallows are carrying a coconut on a line and just happen to fly overhead of Luigi. At the same instant, a blue leader shell is lofted from Bowser. The blue leader shell accidentally clips both African Swallows causing them to drop their coconut. The coconut will fall a distance of  $s(t) = 16t^2$  feet (neglecting common sense, of course), where  $t$  is the time in seconds after the swallows' talons let go of the husk of the coconut.

- If it takes 5 seconds to hit the ground, find the impact velocity.
- Find the acceleration due to gravity.

a.

$$v(t) = 32t$$

$$v(5) = 32(5)$$

$$v(5) = 160 \text{ ft/sec}$$

The impact velocity of the coconut is 160 feet per second. (Obviously the coconut is going down)

b.

$$a(t) = 32 \text{ ft/sec}^2$$

The acceleration due to gravity is 32 feet per second every second.

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### Coconut Shrapnel

#7) Seemingly, Luigi finally catches a break. The two migratory African swallows' coconut narrowly misses Luigi and smashes onto the race track. In what became a brief moment of celebration, Luigi smiles smugly until he realizes coconut shrapnel is headed right towards his lovely, green eyes. Temporarily blinded, Luigi drives off the track. His distance from the track  $t$  seconds after being blinded by coconut shrapnel is  $s(t) = 20t^2 - 2t^3$  feet.

- Find Luigi's distance from the track 4 second after taking coconut shrapnel in the eye.
- Find Luigi's velocity from the track 4 second after taking coconut shrapnel in the eye.
- Find Luigi's acceleration from the track 4 second after taking coconut shrapnel in the eye.

a. 
$$s(4) = 20(4)^2 - 2(4)^3$$

$$= 20(16) - 2(64)$$

$$= 320 - 128$$

$$s(4) = 192 \text{ feet}$$

Four seconds after taking coconut shrapnel in the eye, Luigi's distance from the track is 192 feet.

b. 
$$v(t) = 40t - 6t^2$$

$$v(4) = 40(4) - 6(4)^2$$

$$v(4) = 160 - 6(16)$$

$$= 160 - 96$$

$$v(4) = 64 \text{ ft/sec}$$

Four seconds after taking coconut shrapnel in the eye, Luigi's velocity is 64 feet per second away from the track.

c. 
$$a(t) = 40 - 12t$$

$$a(4) = 40 - 12(4)$$

$$= 40 - 48$$

$$a(4) = -8 \text{ ft/sec}^2$$

Four seconds after taking coconut shrapnel in the eye, Luigi's velocity is decreasing by 8 feet per second every second.

### Luigi's Profit

#8) When the race is over, Luigi finishes in last place, thus not earning the allusive gold trophy. Luigi is furious. He is so angry that instead of signing autographed pictures and trading cards of himself pro bono, he charges his fans for each item. Luigi's profit from selling  $x$  autographed pieces of memorabilia is  $P(x) = 5x^{0.4} - 0.35x^{1.6}$  million gold coins.

- Evaluate  $P(3)$  and interpret your answer.
- Evaluate  $P'(3)$  and interpret your answer.
- Evaluate  $P''(3)$  and interpret your answer.

a. 
$$P(3) = 5(3)^{0.4} - 0.35(3)^{1.6}$$

$$= 5.729387 \text{ million gold}$$

$$P(3) = 5,729,387 \text{ gold}$$

Luigi's profit from selling 3 autographed pieces of memorabilia is 5,729,387 pieces of gold.

b. 
$$P'(x) = 2x^{-0.6} - 0.56x^{0.6}$$

$$P'(3) = 2(3)^{-0.6} - 0.56(3)^{0.6}$$

$$P'(3) = -.048018 \text{ million gold}$$

$$P'(3) = 48,018 \text{ gold/memorabilia}$$

When 3 autographed pieces of memorabilia are sold, Luigi's profit is increasing by 48,018 pieces of gold per piece of memorabilia.