Derivative Applications
Chapter 3 Review

Dirty Inc's Looks

#1) Dirty Inc specializes in selling dirty looks to customers whose faces are too nice to be able to form their own dirty looks. Dirty Inc's profit function is $P(x) = 20\sqrt{x} - 12\sqrt[3]{x}$ dollars, where x is the daily sales of dirty looks

a. Find the marginal profit function.
b. Find the marginal profit when 16 dirty looks have been sold.
c. Interpret your answer from part b.

a. $P(x) = 20x^{\frac{1}{2}} - 12x^{\frac{1}{3}}$

$MP(x) = 10x^{-\frac{1}{2}} - 4x^{-\frac{2}{3}}$

$MP(x) = \frac{10}{\sqrt{x}} - \frac{4}{\sqrt[3]{x^2}}$

b. $MP(16) = \frac{10}{\sqrt{(16)}} - \frac{4}{\sqrt[3]{(16)^2}}$

$= \frac{10}{4} - \frac{4}{\sqrt[3]{256}}$

$= \frac{5}{2} - \frac{4}{\sqrt[3]{256}}$

$MP(16) = \$1.87$ / dirty look

c. When 16 dirty looks have been sold, the total profit is increasing by \$1.87 per dirty look.

or

When 16 dirty looks have been sold, the profit on the next dirty look is \$1.87.

Anti Inc's Jokes

#2) Anit Inc sells jokes by the punchline. There top seller is

Question: "What is red, has large talons, whistles when you squeeze it, and likes to be called Reggie?"

Punchline: "A turtle.
So maybe I lied about it being red… and the talons.
Come to think of it, I lied about the whistling.
And tbh, his name isn't even Reggie.
Yeah, I guess I lied about the whole thing."

Anti Inc's revenue function is $R(x) = 30\sqrt[3]{x} + 4\sqrt{x}$ dollars, where x is the daily sales of punchlines.

a. Find the marginal revenue function.
b. Find the marginal revenue when 32 punchlines have been sold.
c. Interpret your answer from part b.

a. $R(x) = 30x^{\frac{1}{3}} + 4x^{\frac{1}{2}}$

$MR(x) = 10x^{-\frac{2}{3}} + 2x^{-\frac{1}{2}}$

$MR(x) = \frac{10}{\sqrt[3]{x^2}} + \frac{2}{\sqrt{x}}$

b. $MR(32) = \frac{10}{\sqrt[3]{(32)^2}} + \frac{2}{\sqrt{32}}$

$= \frac{10}{\sqrt[3]{1024}} + \frac{2}{\sqrt{32}}$

$R(32) = \$1.35$ / punchline

c. When 32 punchlines have been sold, the total revenue is increasing by \$1.35 per punchline.

or

When 32 punchlines have been sold, the revenue from the next punchline is \$1.35.

Butt Munchers

#3) A growing problem among smokers is their tendency to litter. Scott's entrepreneurial spirit and scientific knowhow has led him to develop a new line of gerbil that will actually munch on the butts of cigarettes. The cigarette butt munchers have a cost of \$5.00 each with fixed costs \$1000 per week.

a. Find the cost function.
b. Find the average cost function.
c. Find the marginal average cost function.
d. Evaluate *MAC(x)* at *x = 10* and interpret your answer.

x = # of butt munchers

a. $C(x) = \$5x + \$1000$

b. $AC(x) = \frac{C(x)}{x}$

$= \frac{5x+1000}{x}$

$AC(x) = 5 + 1000x^{-1}$

c. $MAC(x) = -1000x^{-2}$

$MAC(x) = \frac{-1000}{x^2}$

d. $MAC(10) = \frac{-1000}{(10)^2}$

$= \frac{-1000}{100}$

$MAC(10) = -\$10$ / butt muncher

When 10 butt munchers have been sold, the average cost per butt muncher is decreasing by \$10 per butt munch.

Find the first four derivatives of each function.

#4) $f(x) = 2x^4 + x - 8$ (Use Leibniz)

$\frac{df}{dx} = 8x^3 + 1$

$\frac{d^2f}{dx^2} = 24x^2$

$\frac{d^3f}{dx^3} = 48x$

$\frac{d^4f}{dx^4} = 48$

#5) $f(x) = \sqrt{x^3}$ (Use Newton)

$f(x) = x^{3/2}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} = \frac{3\sqrt{x}}{2}$

$f''(x) = \frac{3}{4}x^{-\frac{1}{2}} = \frac{3}{4\sqrt{x}}$

$f'''(x) = -\frac{3}{8}x^{-3/2} = -\frac{3}{8\sqrt{x^3}}$

$f^{(4)}(x) = \frac{9}{16}x^{-5/2} = \frac{9}{16\sqrt{x^5}}$

#6) If $f(x) = \frac{x^2+1}{2x}$, find $f''(3)$.

$f(x) = \frac{x^2}{2x} + \frac{1}{2x}$ Simplify

$f(x) = \frac{1}{2}x + \frac{1}{2}x^{-1}$

$f'(x) = \frac{1}{2} - \frac{1}{2}x^{-2}$

$f''(x) = x^{-3}$

$f''(x) = \frac{1}{x^3}$

$f''(3) = \frac{1}{(3)^3}$

$f''(3) = \frac{1}{27}$

#7) If $f(x) = \frac{x+1}{x-1}$, find $f''(3)$.

$f'(x) = \frac{(x+1)'(x-1) - (x+1)(x-1)'}{(x-1)^2}$

$= \frac{(1)(x-1) - (x+1)(1)}{(x-1)^2}$

$= \frac{x-1-x-1}{(x-1)^2}$

$f'(x) = \frac{-2}{(x-1)^2}$

$f''(x) = \frac{(-2)'(x-1)^2 - (-2)(x^2-2x+1)'}{[(x-1)^2]^2}$

$= \frac{0(x-1)^2 + 2(2x-2)}{(x-1)^4}$

$= \frac{0 + 2(2x-2)}{(x-1)^4}$

$= \frac{4(x-1)}{(x-1)^4}$

$f''(x) = \frac{4}{(x-1)^3}$

$f''(3) = \frac{4}{[(3)-1]^3}$

$= \frac{4}{[2]^3}$

$= \frac{4}{8}$

$f''(3) = \frac{1}{2}$

#8) If $f(x) = (5x^2 + 3x - 1)(x^2 + 1)$, find the first and second derivative.

$f(x) = 5x^4 + 5x^2 + 3x^3 + 3x - x^2 - 1$ Count it

$f(x) = 5x^4 + 3x^3 + 4x^2 + 3x - 1$

$f'(x) = 20x^3 + 9x^2 + 8x + 3$

$f''(x) = 60x^2 + 18x + 8$

#9) If $f(x) = \frac{1}{x-1}$, find the first and second derivative.

$f'(x) = \frac{(1)'(x-1) - (1)(x-1)'}{(x-1)^2}$

$= \frac{0(x-1) - 1(1)}{(x-1)^2}$

$= \frac{0 - 1}{(x-1)^2}$

$f'(x) = \frac{-1}{(x-1)^2}$

$f''(x) = \frac{(-1)'(x-1)^2 - (-1)(x^2-2x+1)'}{[(x-1)^2]^2}$

$= \frac{(0)(x-1)^2 + 1(2x-2)}{(x-1)^4}$

$= \frac{0 + 2x-2}{(x-1)^4}$

$= \frac{2(x-1)}{(x-1)^4}$

$f''(x) = \frac{2}{(x-1)^3}$

#10) $\frac{d^2}{dr^2}(\pi r^2)|_{r=3}$

$\frac{d}{dr}(\pi r^2) = 2\pi r$

$\frac{d^2}{dr^2}(\pi r^2) = 2\pi$

$\frac{d^2}{dr^2}(\pi r^2)\Big|_{r=3} = 2\pi$

#11) $\frac{d^2}{dr^2}(r^5 + r^4)|_{r=2}$

$\frac{d}{dr}(r^5+r^4) = 5r^4+4r^3$

$\frac{d^2}{dr^2}(r^5+r^4) = 20r^3+12r^2$

$\frac{d^2}{dr^2}(r^5+r^4)\Big|_{r=2} = 20(2)^3+12(2)^2$

$= 20(8) + 12(4)$

$= 160 + 48$

$\frac{d^2}{dr^2}(r^5+r^4)\Big|_{r=2} = 208$

Imagination Population

#12) The population of my imaginary friends t nervous breakdowns from now is predicted to be $p(t) = 12t^{3/2} + 5$ people.

a. Find $p(2)$ and interpret your answer.
b. Find $p'(2)$ and interpret your answer.
c. Find $p''(2)$ and interpret your answer.

$p(t)$ = population

a.

$p(2) = 12\sqrt{(2)}^3 + 5$

$p(2) \approx 39$ imaginary friends

After 2 nervous breakdowns I have about 39 imaginary friends.

b.

$p'(t) = 18t^{1/2}$

$p'(t) = 18\sqrt{t}$

$p'(2) = 18\sqrt{(2)}$

$p'(2) \approx 25$ imaginary friends/breakdown

After 2 nervous breakdowns, my total number of imaginary friends are increasing by 25 friends per breakdown.

c.

$p''(t) = 9t^{-\frac{1}{2}}$

$p''(t) = \frac{9}{\sqrt{t}}$

$p''(2) = \frac{9}{\sqrt{(2)}}$

$p''(2) \approx 6$ imag. friends/breakdown^2

After 2 nervous breakdowns, the population growth rate of my imaginary friends is growing by 6 friends per breakdown each nervous breakdown.

Pizza Rolls

#13) The average time it takes for a 350° Pizza Roll to exit the oven and enter my waiting, salivating mouth is 2.3 seconds. A Pizza Roll's temperature t seconds after burning my tongue is $T(t) = -10\sqrt{t} + 350$ degrees F.

$T(t)$ = Temp of
t = seconds

a. Find $T(4)$ and interpret your answer.
b. Find $T'(4)$ and interpret your answer.
c. Find $T''(4)$ and interpret your answer.

a. $T(4) = -10\sqrt{4} + 350°$

$= -10(2) + 350°$

$= -20 + 350°$

$T(4) = 330°F$

Four seconds after burning my tongue, the pizza roll is 330°F.

b. $T(t) = -10t^{\frac{1}{2}} + 350$

$T'(t) = -5t^{-\frac{1}{2}}$

$T'(t) = \frac{-5}{\sqrt{t}}$

$T'(4) = \frac{-5}{\sqrt{(4)}}$

$T'(4) = \frac{-5}{2}$

$T'(4) = -2.5°F/\text{second}$

Four seconds after burning my mouth, the pizza pocket's temperature is decreasing by 2.5°F per seconds.

c. $T''(t) = \frac{5}{2}t^{-\frac{3}{2}}$

$T''(t) = \frac{5}{2\sqrt{t^3}}$

$T''(4) = \frac{5}{2(\sqrt{4})^3}$

$= \frac{5}{2(2)^3}$

$= \frac{5}{2(8)}$

$T''(4) = \frac{5}{16}\ °F/\text{sec}^2$

Four seconds after burning my mouth, the rate of change of the temperature of the pizza roll is increasing by 5/16 degrees F per second each second.

German Chocolate

#14) A delicious cake is dropped from a reverse albino pigeon while in flight. The height of the moist German chocolate cake after t seconds is $s(t) = 75 - 16t^2$ feet (neglecting air resistance, obviously).

a. How long will it take the German chocolate cake to reach the ground?
b. What will the velocity of the cake be when it impacts the ground?

$s(t)$ = height
t = seconds

a. $0 = 75 - 16t^2$

$16t^2 = 75$

$t^2 = \frac{75}{16}$

$t = \pm\sqrt{\frac{75}{16}}$ (negative makes no sense)

$t = 2.2$ seconds

The moist German chocolate cake will take about 2.2 seconds to hit the ground.

b. $s(t) = 75 - 16t^2$

$v(t) = -32t$

$v(2.2) = -32(2.2)$

$v(2.2) = 70.4$ ft/sec

The moist chocolate cake will hit the ground with an impact velocity of 70.4 feet per second.

The cake is a lie.

Cat-a-pterodactyl

#15) A evolutionary cat-a-pterodactyl (yes, it's exactly what you think it is) is carrying a newt. The cat-a-pterodactyl is planning on dropping the newt on an unsuspecting dog-a-saurus (also, what you think it is.) The newt will fall a distance of $s(t) = 16t^2$ feet (neglecting all logic and reasoning, of course). Please note *t* is the time in seconds after the cat-a-pterodactyl's talons/paws let go of the flesh of the newt.

a. If it takes 4 seconds to hit the dog-a-saurus, find the impact velocity.
b. Find the acceleration due to gravity.

a. $v(t) = 32t$

$v(4) = 32(4)$

$v(4) = 128$ ft/sec

Four seconds after the cat-a-pterodactyl dropped the newt, the impact velocity of the newt on the the dog-a-saurus' noggin is 128 feet per second.

b. $a(t) = 32$ ft/sec²

The acceleration due to gravity of the falling newt is 32 feet per second each second.

#16) P(x) = total profit from selling *x* blocks of head
x = number of blocks of head

Interpret P(3) = ~~$~~21

After selling 3 block heads, the total profit is $21.

Interpret MP(3) = ~~$~~8/block head (Give two interpretations)

After 3 block heads are sold, the total profit is increasing by $8 per block head sold.

Or

After 3 block heads are sold, the profit from selling the next block head is $8.

Interpret AP(3) = ~~$~~7

After selling 3 block heads, the average profit is $7 per block head.

Interpret MAP(70) = ~~$~~1/block head

After selling 70 block heads the average profit per block head is increasing by $1 per block head sold.