Advanced Derivative Rules
4.1A – The Chain Rule

A: Decompose the functions by finding functions $h(x)$ and $k(x)$ such that the following expression is the composition $h(k(x))$.

#1) $(x^5 - 5x^2)^{10}$

inside $= K(x) = x^5 - 5x^2$
outside $= h(x) = x^{10}$

#2) $\sqrt{9x^2 + 3}$

inside $= K(x) = 9x^2 + 3$
outside $= h(x) = \sqrt{x}$

#3) $\frac{1}{x^5 - 7x + 1}$

inside $= K(x) = x^5 - 7x + 1$
outside $= h(x) = x^{-1}$

#4) $\sqrt{x^2 + 1} - 6$

inside $= K(x) = x^2 + 1$
outside $= h(x) = \sqrt{x} - 6$

B: Use the Chain Rule to find the derivative of each function.

#5) $g(x) = (x^4 + 1)^2$

$g'(x) = 2(x^4+1)^1 \cdot (x^4+1)'$
$g'(x) = 2(x^4+1)(4x^3)$
$g'(x) = 8x^3(x^4+1)$

#6) $h(x) = (3x^2 - 7x + 5)^3$

$h'(x) = 3(3x^2-7x+5)^2 \cdot (3x^2-7x+5)'$
$h'(x) = 3(3x^2-7x+5)^2(6x-7)$

#7) $k(x) = (2x^4 + 6x^2 + 10)^{12}$

$K'(x) = 12(2x^4+6x^2+10)^{11}(2x^4+6x^2+10)'$
$K'(x) = 12(2x^4+6x^2+10)^{11}(8x^3+12x)$

#8) $f(x) = \sqrt{x^6 + 1}$

$f'(x) = \frac{1}{2}(x^6+1)^{-\frac{1}{2}}(x^6+1)'$

$f'(x) = \frac{6x^5}{2\sqrt{x^6+1}}$

#9) $y = \sqrt{7x - 1}$

$y' = \frac{1}{2}(7x-1)^{-\frac{1}{2}}(7x-1)'$

$y' = \frac{1}{2\sqrt{7x-1}}(7)$

$y' = \frac{7}{2\sqrt{7x-1}}$

#10) $y = (x^2 + 1)^{4/5}$

$y' = \frac{4}{5}(x^2+1)^{-1/5}(x^2+1)'$

$y' = \frac{4}{5}(x^2+1)^{-1/5}(2x)$

$y' = \frac{8x}{5\sqrt[5]{x^2+1}}$

#11) $y = \left(\frac{1}{x^2+1}\right)^5 = (x^2+1)^{-5}$

$y' = -5(x^2+1)^{-6}(x^2+1)'$

$y' = \frac{-5}{(x^2+1)^6}(2x)$

$y' = \frac{-5(2x)}{(x^2+1)^6}$

$y' = \frac{-10x}{(x^2+1)^6}$

#12) $y = \frac{1}{\sqrt{8x-1}} = (8x-1)^{-\frac{1}{2}}$

$y' = -\frac{1}{2}(8x-1)^{-3/2}(8x-1)'$

$y' = \frac{-1}{2\sqrt{(8x-1)^3}}(8)$

$y' = \frac{-4}{\sqrt{(8x-1)^3}}$

#13) $y = \frac{1}{\sqrt{2x+1}} = (2x+1)^{-1/2}$

$y' = -\frac{1}{2}(2x+1)^{-3/2}(2x+1)'$

$y' = \frac{-1}{2\sqrt{(2x+1)^3}}(2)$

$y' = \frac{-1}{\sqrt{(2x+1)^3}}$

George's Jimmy Crack Corn Inc

#14) George is an idiot. Everyone knows it. Even he knows it. After hearing the folk song "Jimmy Crack Corn," George decides to start a business venture selling cracked corn. His newly founded company, *George's Jimmy Crack Corn Inc* has a cost function of $C(x) = \sqrt{5x^2 + 1}$ dollars, where x is the number of cracked corn made.

x= cracked corns

a. Find the marginal cost function
b. Evaluate $MC(x)$ when 100 cracked corns have been produced and interpret your answer.

a. $C(x) = (5x^2+1)^{\frac{1}{2}}$

$MC(x) = \frac{1}{2}(5x^2+1)^{-\frac{1}{2}}(5x^2+1)'$

$= \frac{1}{2\sqrt{5x^2+1}}(10x)$

$MC(x) = \frac{5x}{\sqrt{5x^2+1}}$

b. $MC(100) = \frac{5(100)}{\sqrt{5(100)^2+1}}$

$= \frac{500}{\sqrt{5(10000)+1}}$

$= \frac{500}{\sqrt{50001}}$

$MC(100) \approx 2.34$

When 100 cracked corns have been produced the next cracked corn will cost $2.34.

OR

When 100 cracked corns have been produced the total cost is increasing by $2.34 per cracked corn.

Mutual Funds

#15) With his newfound business dragging him into a financial gutter, George decides to finally do something smart by investing money in mutual funds. However, because he doesn't have any money to invest, he plans on stealing money from the Make-A-Wish Foundation. Through delusion and blatant stupidity, George assumes he'll be able to steal $5000. His plan is to then invest into a mutual fund paying r% in returns annually. In 10 years the value of the mutual fund will be $V(r) = 5000(1 + 0.01r)^{10}$ dollars.

V= $ value
r= % return

a. Find V(12) and interpret your answer.
b. Find $V'(12)$ and interpret your answer.

a. $V(12) = 5000(1+0.01(12))^{10}$

$= 5000(1+0.12)^{10}$

$= 5000(1.12)^{10}$

$V(12) \approx \$15{,}529.24$

If George's mutual fund can earn 12% annually, his $5000 will grow to $15,529.24 in 10 years.

b. $V(r) = 5000(1+0.01r)^{10}$

$V'(r) = 50{,}000(1+0.01r)^{9}\cdot(1+0.01r)'$

$= 50{,}000(1+0.01r)^{9}(0.01)$

$V'(r) = 500(1+0.01r)^{9}$

$V'(12) = 500[1+0.01(12)]^{9}$

$= 500[1+0.12]^{9}$

$= 500[1.12]^{9}$

$V'(12) \approx \$1386/\%$

When the rate of return is 12%, the annual return will increase by $1386 per year for each 1% increase in rate of return.

Tusky's Population

#16) After raiding the Make-A-Wish Foundation searching for $5000, all George could find was a basket full of wishes. Depressed at failing again, he decided to move out of Tuscarawas and venture to California. The would-be inhabitants of Tusky rejoiced at him vacating their beloved homeland and started moving back to the village. The population of Tusky $\boxed{x \text{ years from George leaving}}$ it is expected to be $P(x) = \sqrt{x^2 - 7}$ hundred people for $1 \leq x \leq 5$.

a. Find $P(4)$ and interpret your answer.
b. Find $P'(4)$ and interpret your answer.

$P(x)$ = people in hundred
x = years

a. $P(4) = \sqrt{(4)^2 - 7}$
$= \sqrt{16-7}$
$= \sqrt{9}$
$P(4) = 3$ hundred

Four years after George leaves Tusky, the population of Tusky is 300 people.

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b. $P(x) = (x^2-7)^{1/2}$

$P'(x) = \frac{1}{2}(x^2-7)^{-\frac{1}{2}}\,(x^2-7)'$

$= \frac{1}{2\sqrt{x^2-7}}(2x)$

$P'(x) = \frac{x}{\sqrt{x^2-7}}$

$P'(4) = \frac{4}{\sqrt{(4)^2-7}}$

$= \frac{4}{\sqrt{16-7}}$

$= \frac{4}{\sqrt{9}}$

$= \frac{4}{3}$ hundred people/year

$P'(4) \approx 133$ people/year

Four years after George left Tusky, the population of Tusky is increasing by about 133 people per year.