Advanced Derivative Rules

4.2C – Products, Quotients & Trigonometry II

A: Differentiate using the Product Rule

#1) $\frac{d}{dx}\left(\frac{1}{3}x^3 \cot(x)\right)$

$$= \left(\frac{1}{3}x^3\right)' \cot(x) + \left(\frac{1}{3}x^3\right)\left[\cot(x)\right]'$$

$$= x^2 \cot(x) + \frac{1}{3}x^3\left(-\csc^2(x)\right)$$

$$= x^2 \cot(x) - \frac{1}{3}x^3 \csc^2(x)$$

#2) If $f(x) = \frac{1}{5}x^5 \sec(x)$, find $f'(x)$.

$$f'(x) = \left[\frac{1}{5}x^5\right)' \sec(x) + \frac{1}{5}x^5\left[\sec(x)\right]'$$

$$= x^4 \sec(x) + \frac{1}{5}x^5 \sec(x)\tan(x)$$

#3) $\frac{d}{dx}[x\cos(x)]$

$$= x' \cdot \cos(x) + x\left[\cos(x)\right]'$$

$$= \cos(x) - x\sin(x)$$

#4) If $f(x) = (\sin(x) + 1)\tan(x)$, find $f'(x)$.

$$f'(x) = \left[\sin(x) + 1\right]' \tan(x) + \left[\sin(x) + 1\right]\left[\tan(x)\right]'$$

$$= \cos(x)\tan(x) + \left[\sin(x) + 1\right]\sec^2(x)$$

$$= \cos(x) \cdot \frac{\sin(x)}{\cos(x)} + \left[\sin(x) + 1\right]\sec^2(x)$$

$$= \sin(x) + \left[\sin(x) + 1\right]\sec^2(x)$$

#5) $\frac{d}{dx}[x\sin(x)\cos(x)\sec(x)\cot(x)]$

$$= \frac{d}{dx}\left[x\cancel{\sin(x)}\,\cancel{\cos(x)}\,\frac{1}{\cancel{\cos(x)}}\,\frac{\cos(x)}{\cancel{\sin(x)}}\right]$$

$$= \frac{d}{dx}\left[x\cos(x)\right]$$

$$= \frac{d}{dx}x \cdot \cos(x) + x\,\frac{d}{dx}\cos(x)$$

$$= \cos(x) - x\sin x$$

#6) If $f(x) = \frac{1}{2}\csc(x)$, find $f'(x)$.

$$f'(x) = -\frac{1}{2}\csc(x)\cot(x)$$

#7) $\frac{d}{dx}\left[\frac{1}{x}\cancel{\tan(x)}\sin(x)\cancel{\cot(x)}\right]$

$$= \frac{d}{dx}\left[\frac{1}{x}\sin(x)\right] = \frac{d}{dx}\left(\frac{\sin(x)}{x}\right)$$

$$= \frac{\frac{d}{dx}\sin(x) \cdot x - \sin(x)\frac{d}{dx}(x)}{(x)^2}$$

$$= \frac{\cos(x) \cdot x - \sin(x) \cdot 1}{x^2}$$

$$= \frac{x\cos(x) - \sin(x)}{x^2}$$

#8) Prove that $\frac{d}{dx}\tan(x) = \sec^2(x)$

$$\frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right] = \frac{\left[\sin(x)\right]'\cos(x) - \sin(x)\left[\cos(x)\right]'}{\left[\cos(x)\right]^2}$$

$$= \frac{\cos(x) \cdot \cos(x) - \sin(x)\left(-\sin(x)\right)}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)}$$

$$= \sec^2(x)$$

B: Differentiating using the Quotient Rule.

#9) $\frac{d}{dx}\left(\frac{\cos(x)}{x^2}\right)$

$$= \frac{\frac{d}{dx}[\cos(x)]\cdot x^2 - \cos(x)\frac{d}{dx}[x^2]}{[x^2]^2}$$

$$= \frac{-\sin(x)\cdot x^2 - \cos(x)\cdot 2x}{x^4}$$

$$= \frac{-x^2\sin(x) - 2x\cos(x)}{x^4}$$

$$= \frac{-x\sin(x) - 2\cos(x)}{x^3}$$

#10) $\frac{d}{dx}\left(\frac{6x^4}{\cos(-x)}\right) = \frac{d}{dx}\left[\frac{6x^4}{\cos(x)}\right]$

$$= \frac{\frac{d}{dx}(6x^4)\cdot\cos(x) - 6x^4\frac{d}{dx}[\cos(x)]}{[\cos(x)]^2}$$

$$= \frac{24x^3\cos(x) - 6x^4[-\sin(x)]}{\cos^2(x)}$$

$$= \frac{24x^3\cos(x) + 6x^4\sin(x)}{\cos^2(x)}$$

#11) $\frac{d}{dx}\left(\frac{x}{\csc(x)}\right)$

$$= \frac{\frac{d}{dx}(x)\cdot\csc(x) - x\cdot\frac{d}{dx}\csc(x)}{[\csc(x)]^2}$$

$$= \frac{1\cdot\csc(x) - x(-\csc(x)\cot(x))}{\csc^2(x)}$$

$$= \frac{\csc(x) + x\csc(x)\cot(x)}{\csc^2(x)}$$

$$= \frac{\csc(x)[1 + x\cot(x)]}{\csc^2(x)}$$

$$= \frac{1 + x\cot(x)}{\csc(x)}$$

#12) $\frac{d}{dx}\left(\frac{\sec(x)}{x^2-25}\right)$

$$= \frac{[\sec(x)]'(x^2-25) - \sec(x)[x^2-25]'}{[x^2-25]^2}$$

$$= \frac{\sec(x)\tan(x)(x^2-25) - \sec(x)(2x)}{[x^2-25]^2}$$

$$= \frac{(x^2-25)\sec(x)\tan(x) - 2x\sec(x)}{(x^2-25)^2}$$

#13) $\frac{d}{dx}\left(\frac{\tan(x)+\sin(x)}{\sqrt{x^3}}\right)$

$$= \frac{[\tan(x)+\sin(x)]'\sqrt{x^3} - [\tan(x)+\sin(x)](x^{\frac{3}{2}})'}{(\sqrt{x^3})^2}$$

$$= \frac{[\sec^2(x)+\cos(x)]\sqrt{x^3} - [\tan(x)+\sin(x)]\frac{3}{2}x^{\frac{1}{2}}}{x^3}$$

#14) $\frac{d}{dx}\left(\frac{\cos(x)}{\sin(x)}\right) = \frac{d}{dx}\cot(x)$

$$= -\csc^2(x)$$