# Advanced Derivative Rules
## 4.2 – Derivatives & Trigonometry

**Derivatives of the Six Trigonometric Functions**

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

**Basic Identities**

Reciprocal Identities: The following trig identities hold for all values of *A* except those for which any function is undefined.

$$\sin(x) = \frac{1}{\csc(x)} \qquad \csc(x) = \frac{1}{\sin(x)}$$

$$\cos(x) = \frac{1}{\sec(x)} \qquad \sec(x) = \frac{1}{\cos(x)}$$

$$\tan(x) = \frac{1}{\cot(x)} \qquad \cot(x) = \frac{1}{\tan(x)}$$

Quotient Identities: The following trig identities hold for all values of *A* except those for which any function is undefined.

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

Pythagorean Identities: The following trig identities hold for all values of *A* except those for which any function is undefined.

$$\sin^2(x) + \cos^2(x) = 1$$

$$\tan^2(x) + 1 = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

**Even & Odd Trig Functions**

Recall that an even function is said to be even if its symmetric with respect to the y-axis, f(x) = f(-x).

$$\cos(-x) = \cos(x)$$

Recall that an odd function is said to be odd if it is symmetric with respect to the origin,–f(x) = f(-x).

$$\sin(-x) = -\sin(x)$$

$$\tan(-x) = -\tan(x)$$

Ex A: Differentiating using the Chain Rule.

#1) $\frac{d}{dx}[\sin^4(x)] = \frac{d}{dx}[\sin(x)]^4$

$= 4[\sin(x)]^3 \cdot \frac{d}{dx}\sin(x)$

$= 4\sin^3(x)\cdot\cos(x)\cdot x'$

$= 4\sin^3(x)\cos(x)\cdot(1)$

$= 4\sin^3(x)\cos(x)$

#2) $\frac{d}{dx}[\sin(x^4)]$

$= \cos(x^4)\ \frac{d}{dx}x^4$

$= \cos(x^4)\cdot x^3$

$= x^3\cos(x^4)$

#3) $\frac{d}{dx}[\cos^4(x)] = \frac{d}{dx}[\cos(x)]^4$

$= 4[\cos(x)]^3\cdot\frac{d}{dx}\cos(x)$

$= 4\cos^3(x)\ (-\sin(x))\cdot x'$

$= -4\cos^3(x)\sin(x)\ (1)$

$= -4\cos^3(x)\sin(x)$

#4) $\frac{d}{dx}[\cos(x^4)]$

$= \sin(x^4)\cdot\frac{d}{dx}(x^4)$

$= \sin(x^4)\cdot 4x^3$

$= 4x^3\sin(x^4)$

Ex B: Differentiating using the Product Rule.

#5) $\frac{d}{dx}(x^4\sin(x)) = \frac{d}{dx}x^4\cdot\sin(x) + x^4\frac{d}{dx}\sin(x)$

$= 4x^3\sin(x) + x^4\cos(x)$

#6) If $f(x) = \sin(x)\sec(x)$, find $f'(x)$.

$= \sin(x) \cdot \frac{1}{\cos(x)}$

$= \frac{\sin(x)}{\cos(x)}$

$f(x) = \tan(x)$

$f'(x) = \sec^2(x)$

Ex C: Differentiating using the Quotient Rule.

#7) If $y = \frac{x^2}{\sin(x)}$, find $y'$.

$y' = \frac{[x^2]'\sin(x) - x^2[\sin(x)]'}{[\sin(x)]^2}$

$y' = \frac{2x \cdot \sin(x) - x^2\cos(x)}{\sin^2(x)}$

#8) Use the identity $\cot(x) = \frac{\cos(x)}{\sin(x)}$ to prove $\frac{d}{dx}\cot(x) = -\csc^2(x)$

$\frac{d}{dx}\cot(x) = \frac{d}{dx}\frac{\cos(x)}{\sin(x)}$

$= \frac{\frac{d}{dx}\cos(x) \cdot \sin(x) + \cos(x) \cdot \frac{d}{dx}(\sin(x))}{[\sin(x)]^2}$

$= \frac{-\sin(x)\sin(x) - \cos(x)\cos(x)}{\sin^2(x)}$

$= \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)}$

$= \frac{-1[\sin^2(x) + \cos^2(x)]}{\sin^2(x)}$

$= \frac{-1 \cdot 1}{\sin^2(x)}$

$\frac{d}{dx}\cot(x) = -\csc^2(x)$

#9) Differentiating using Regret, Sadness and a heavy dose of Eraser.

If $y = \frac{\tan(x)}{\sin(x)}$, find $y'$.

$y = \frac{\frac{\cancel{\sin(x)}}{\cos(x)}}{\cancel{\sin(x)}} \cdot \frac{\frac{1}{\cancel{\sin(x)}}}{\frac{1}{\cancel{\sin(x)}}}$

$y = \frac{1}{\cos(x)}$

$y = \sec(x)$

$y' = \sec(x)\tan(x)$