Advanced Derivative Rules
4.3A – Differentiating Using Two Rules

A: Find the derivative of each function. Factor answers when appropriate.

#1) $h(x) = [(x^3 - x)^5 + x^2]^2$

$h'(x) = 2[(x^3-x)^5 + x^2]^1 \cdot [(x^3-x)^5 + x^2]'$

$h'(x) = 2[(x^3-x)^5 + x^2]\,[5(x^3-x)^4 (x^3-x)' + 2x]$

$h'(x) = 2[(x^3-x)^5 + x^2]\,[5(x^3-x)^4 (3x^2-1) + 2x]$

#2) $h(x) = [(x-1)^2 + 5x]^7$

$h'(x) = 7[(x-1)^2 + 5x]^6 \cdot [(x-1)^2 + 5x]'$

$h'(x) = 7[(x-1)^2 + 5x]^6 \cdot [2(x-1)^1 \cdot (x-1)' + 5]$

$h'(x) = 7[(x-1)^2 + 5x]^6 \cdot [(2x-2) \cdot (1) + 5]$

$h'(x) = 7[(x-1)^2 + 5x]^6 \cdot [2x + 3]$

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#3) $y = 4x^2(9x-1)^3$

$$y' = [4x^2]'(9x-1)^3 + 4x^2 \cdot [(9x-1)^3]'$$

$$y' = 8x \cdot (9x-1)^3 + 4x^2 \cdot 3(9x-1)^2 (9x-1)'$$

$$y' = 8x \cdot (9x-1)^3 + 4x^2 \cdot 3(9x-1)^2 (9)$$

$$y' = 4x(9x-1)^2 [2(9x-1) + x \cdot 3 \cdot 9]$$

$$y' = 4x(9x-1)^2 [18x - 2 + 27x]$$

$$y' = 4x(9x-1)^2 [45x - 2]$$

#4) $h(x) = (x^2+1)^4(6x-1)^5$

$$h'(x) = [(x^2+1)^4]'(6x-1)^5 + (x^2+1)^4[(6x-1)^5]'$$

$$h'(x) = 4(x^2+1)^3(x^2+1)'(6x-1)^5 + (x^2+1)^4 5(6x-1)^4 (6x-1)'$$

$$h'(x) = 4(x^2+1)^3(2x)(6x-1)^5 + (x^2+1)^4 5(6x-1)^4 (6)$$

$$h'(x) = 2(x^2+1)^3(6x-1)^4 [4x(6x-1) + (x^2+1)5 \cdot 3]$$

$$h'(x) = 2(x^2+1)^3(6x-1)^4 [24x^2 - 4x + 15x^2 + 15]$$

$$h'(x) = 2(x^2+1)^3(6x-1)^4 [39x^2 - 4x + 15]$$

#5) $y = \sin(2x)\ \cos^2(x)$

$y' = [\sin(2x)]' \cdot \cos^2(x) + \sin(2x) \cdot [(\cos(x))^2]'$

$y' = \cos(2x) \cdot (2x)' \cdot \cos^2(x) + \sin(2x) \cdot 2\cos(x) \cdot [\cos(x)]'$

$y' = \cos(2x) \cdot (2) \cdot \cos^2(x) + \sin(2x) \cdot 2\cos(x) \cdot (-\sin(x))\ (x)'$

$y' = \cos(2x) \cdot (2) \cdot \cos^2(x) + \sin(2x) \cdot 2\cos(x) \cdot (-\sin(x))\ (1)$

$y' = 2\cos(2x)\cos^2(x) - 2\sin(2x)\cos(x)\sin(x)$

#6) $y = \sin^9(x)\cos^3(x)$

$y' = [\sin^9(x)]' \cos^3(x) + \sin^9(x)[\cos^3(x)]'$

$y' = 9\sin^8(x) \cdot [\sin(x)]' \cos^3(x) + \sin^9(x) \cdot 3\cos^2(x)\ [\cos(x)]'$

$y' = 9\sin^8(x) \cdot \cos(x) \cdot \cos^3(x) + \sin^9(x) \cdot 3\cos^2(x)\ (-\sin(x))$

$y' = 9\sin^8(x)\cos^4(x) - 3\sin^{10}(x)\cos^2(x)$

$y' = 3\sin^8(x)\cos^2(x)\left[3\cos^2(x) - \sin^2(x)\right]$

#7) $y = \frac{\cos^2(x)}{\cot^2(x)}$

$$= \frac{\cancel{\cos^2(x)}}{\frac{\cancel{\cos^2(x)}}{\cancel{\sin^2(x)}}} \cdot \frac{\sin^2(x)}{\cancel{\sin^2(x)}}$$

$$y = \sin^2(x)$$

$$y' = 2\sin(x) \cdot [\sin(x)]'$$

$$y' = 2\sin(x)\cos(x)$$

#8) $y = -\tan^2(55x) + \sec^2(55x) + \sin^2(x)$

$$y = -\tan^2(55x) + [1 + \tan^2(55x)] + \sin^2(x)$$

$$y = 1 + \sin^2(x)$$

$$y' = 0 + 2\sin(x) \cdot [\sin(x)]'$$

$$y' = 2\sin(x)\cos(x)$$

#9) $y = 2\sin(x)\cot(x)\sec(x) + 2\tan^2 x$

$$y = 2\cancel{\sin(x)} \cdot \frac{\cancel{\cos(x)}}{\cancel{\sin(x)}} \cdot \frac{1}{\cancel{\cos(x)}} + 2\tan^2(x)$$

$$y = 2 + 2\tan^2(x)$$

$$y' = 0 + 4\tan(x) \cdot [\tan(x)]'$$

$$y' = 4\tan(x)\sec^2(x)$$

#10) $y = \sin(x)\,[\sin(x) + \cos(x)] + \cos^2(x) - \frac{\tan^2(x)+1}{\frac{1}{\cos^2(x)}}$

$$y = \sin^2(x) + \sin(x)\cos(x) + \cos^2(x) - \frac{\sec^2(x)}{\sec^2(x)}$$

$$y = 1 + \sin(x)\cos(x) - 1$$

$$y = \sin(x)\cos(x)$$

$$y' = [\sin(x)]'\cos(x) + \sin(x)[\cos(x)]'$$

$$y' = \cos(x)\cos(x) + \sin(x)[-\sin(x)]$$

$$y' = \cos^2(x) - \sin^2(x)$$

George's Headache

#11) While traveling by donkey to California, George's body takes a beating. He starts to develop a headache. Trying to gain some relief, he pops a couple of Advil. The strength of George's reaction to a dose of x milligrams of Advil is $R(x) = 2x\sqrt{10 - \frac{1}{2}x}$ for $0 \le x \le 20$. If $R'(x)$ is called the sensitivity of the Advil, find George's sensitivity to the Advil for a dose of 10 mg. (Use a sentence answer.)

$$R'(x) = (2x)'\left(10-\tfrac{1}{2}x\right)^{\frac{1}{2}} + 2x\left[\left(10-\tfrac{1}{2}x\right)^{\frac{1}{2}}\right]'$$

$$= 2\left(10-\tfrac{1}{2}x\right)^{\frac{1}{2}} + 2x\left(\tfrac{1}{2}\right)\left(10-\tfrac{1}{2}x\right)^{-\frac{1}{2}}\left(10-\tfrac{1}{2}x\right)'$$

$$= 2\left(10-\tfrac{1}{2}x\right)^{\frac{1}{2}} + x\left(10-\tfrac{1}{2}x\right)^{-\frac{1}{2}}\left(-\tfrac{1}{2}\right)$$

GCF: $\frac{1}{2}\left(10-\frac{1}{2}x\right)^{-\frac{1}{2}}$

$$= \tfrac{1}{2}\left(10-\tfrac{1}{2}x\right)^{-\frac{1}{2}}\left[4\left(10-\tfrac{1}{2}x\right) - x\right]$$

$$= \frac{1}{2\sqrt{10-\frac{1}{2}x}}\left[40-2x-x\right]$$

$$R'(x) = \frac{40-3x}{2\sqrt{10-\frac{1}{2}x}}$$

$$R'(10) = \frac{40-3(10)}{2\sqrt{10-\frac{1}{2}(10)}}$$

$$= \frac{40-30}{2\sqrt{10-5}}$$

$$= \frac{10}{2\sqrt{5}}$$

$$= \frac{5}{\sqrt{5}}$$

$$R'(10) \approx 2.2$$

George's sensitivity to 10 mg of Advil is 2.2.