A: Find the derivative of each function. Factor answers when appropriate.

#1) $y = \left(\frac{x+1}{x-1}\right)^2$

$y' = 2\left(\frac{x+1}{x-1}\right)^1 \cdot \left(\frac{x+1}{x-1}\right)'$ CHAIN

$y' = \frac{2(x+1)}{x-1} \cdot \frac{(x+1)'(x-1)-(x+1)(x-1)'}{(x-1)^2}$ Quotient

$= \frac{2(x+1)}{x-1} \cdot \frac{(1)(x-1)-(x+1)(1)}{(x-1)^2}$

$= \frac{2(x+1)}{x-1} \cdot \frac{x-1-x-1}{(x-1)^2}$

$= \frac{2(x+1)}{x-1} \cdot \frac{-2}{(x-1)^2}$

$y' = \frac{-4(x+1)}{(x-1)^3}$

#2) $y = \left(\frac{2x^3}{x-1}\right)^5$

$y' = 5\left(\frac{2x^3}{x-1}\right)^4 \cdot \left(\frac{2x^3}{x-1}\right)'$ CHAIN

$y' = 5\,\frac{16x^{12}}{(x-1)^4} \cdot \frac{(2x^3)'(x-1)-2x^3(x-1)'}{(x-1)^2}$ QUOTIENT

$= \frac{80x^{12}}{(x-1)^4} \cdot \frac{6x^2(x-1)-2x^3(1)}{(x-1)^2}$

$= \frac{80x^{12}}{(x-1)^4} \cdot \frac{6x^3-6x^2-2x^3}{(x-1)^2}$

$= \frac{80x^{12}}{(x-1)^4} \cdot \frac{4x^3-6x^2}{(x-1)^2}$

$= \frac{80x^{12}}{(x-1)^4} \cdot \frac{2x^2(2x-3)}{(x-1)^2}$

$y' = \frac{160x^{14}(2x-3)}{(x-1)^6}$

#3) $y = \sqrt{3 + \sqrt{x}}$

$y = (3 + x^{\frac{1}{2}})^{\frac{1}{2}}$

$y' = \frac{1}{2}(3+\sqrt{x})^{-\frac{1}{2}} \cdot (3+x^{\frac{1}{2}})'$ CHAIN

$y' = \frac{1}{2\sqrt{3+\sqrt{x}}}\left(0+\frac{1}{2}x^{-\frac{1}{2}}\right)$

$y' = \frac{1}{2\sqrt{3+\sqrt{x}}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$y' = \frac{1}{4\sqrt{x}\sqrt{3+\sqrt{x}}}$

#4) $f(x) = \sqrt{x - 4 + \sqrt{x^3}}$

$f(x) = (x-4+x^{\frac{3}{2}})^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x-4+\sqrt{x^3})^{-\frac{1}{2}} \cdot (x-4+x^{\frac{3}{2}})'$ CHAIN

$= \frac{1}{2\sqrt{x-4+\sqrt{x^3}}} \cdot \left(1-0+\frac{3}{2}x^{\frac{1}{2}}\right)$

$= \frac{1}{2\sqrt{x-4+\sqrt{x^3}}} \cdot \left(1+\frac{3}{2}\sqrt{x}\right)$

$= \frac{\left(1+\frac{3}{2}\sqrt{x}\right)\cdot 2}{\left(2\sqrt{x-4+\sqrt{x^3}}\right)\cdot 2}$

$f'(x) = \frac{2+3\sqrt{x}}{4\sqrt{x-4+\sqrt{x^3}}}$

#5) $y = \sin(x^2)\ \cos^2(x)$

$y' = [\sin(x^2)]' \cos^2(x) + \sin(x^2)\left([\cos(x)]^2\right)'$

$y' = \cos(x^2)(x^2)' \cos^2(x) + \sin(x^2)\, 2[\cos(x)]\, [\cos(x)]'$

$y' = 2x\cos(x^2)\cos^2(x) + \sin(x^2)\, 2\cos(x)\, (-\sin(x))$

$y = 2x\cos(x^2)\cos^2(x) - 2\sin(x^2)\cos(x)\sin(x)$

#6) $y = \sin^2(x)\cos^2(x)$

$y' = \left([\sin(x)]^2\right)' \cos^2(x) + \sin^2(x)\left([\cos(x)]^2\right)'$

$= 2(\sin(x))(\sin(x))' \cos^2(x) + \sin^2(x)\, 2(\cos(x))(\cos(x))'$

$= 2\sin(x)\cos(x)\cos^2(x) + \sin^2(x)\, 2\cos(x)(-\sin(x))$

$= 2\sin(x)\cos^3(x) - 2\sin^3(x)\cos(x)$

#7) $y = \frac{\sin^2(x)}{\tan^2(x)}$

$$y = \frac{\sin^2(x)}{\frac{\sin^2(x)}{\cos^2(x)}}$$

$$y = \sin^2(x) \cdot \frac{\cos^2(x)}{\sin^2(x)}$$

$$y = \cos^2(x)$$

$$y' = 2\cos(x)\,(-\sin(x))$$

$$y' = -2\cos(x)\sin(x)$$

#8) $y = -\cos^2(x^2) + \csc^2(x^2) - \sin^2(x^2)$

$$y = -\cos^2(x^2) - \sin^2(x^2) + \csc^2(x^2)$$

$$y = -\left[\cos^2(x^2) + \sin^2(x^2)\right] + \csc^2(x^2)$$

$$y = -\left[\quad 1 \quad\right] + \csc^2(x^2)$$

$\boxed{1 + \cot^2(x) = \csc^2(x) \quad \cot^2(x) = -1 + \csc^2(x)}$

$$y = -1 + \csc^2(x^2)$$

$$y = \cot^2(x^2)$$

$$y' = 2\cot(x^2) \cdot [\cot(x^2)]'$$

$$y' = 2\cot(x^2)(-\csc^2(x^2)) \cdot (x^2)'$$

$$y = -2\cot(x^2)\csc^2(x^2) \cdot 2x$$

$$y' = -4x\cot(x^2)\csc^2(x^2)$$

#9) $y = 2\cos(x)\cot(x)\csc(x)2\tan^2 x$

$$y = 2\cancel{\cos(x)} \cdot \frac{\cancel{\cos(x)}}{\cancel{\sin(x)}} \cdot \frac{1}{\cancel{\sin(x)}} \cdot 2\frac{\cancel{\sin^2(x)}}{\cancel{\cos^2(x)}}$$

$$y = 4$$

$$y' = 0$$

#10) $y = -\tan(x)\,[\tan(x) + \cot(x)] + \sec^2(x) - \frac{\cot^2(x)+1}{\frac{1}{\sin^2(x)}}$

$$y = -\tan^2(x) - 1 + \sec^2(x) - \frac{\csc^2(x)}{\csc^2(x)}$$

$$= -\left[\tan^2(x) + 1\right] + \sec^2(x) - 1$$

$$= -\left[\sec^2(x)\right] + \sec^2(x) - 1$$

$$= -1$$

$$y' = 0$$

Jimmy Dean's Sausage

#11) Tired, hungry, and frustrated, George pulls off the road and dismounts his donkey. He sits down in disgust as the hot sun beats down on him. Meanwhile, a Jimmy Dean's Sausage truck traveling down the road accidently spills hundreds of packages of Jimmy Dean's Sausage out of the truck. Excitedly, George shoves several packages of uncooked sausage into his mouth. Moments later George becomes ill. The strength of George's allergic reaction to trichinosis in x pounds of Jimmy Dean's Sausage is $A(x) = 3x\sqrt{20 - \frac{1}{4}x}$ for certain values of x. If $A'(x)$ is called the sensitivity of the reaction, find George's sensitivity to the sausage after a 5 pound binge. (Use a sentence answer.)

$$A(x) = 3x\left(20 - \tfrac{1}{4}x\right)^{\frac{1}{2}}$$

$$A'(x) = (3x)'\left(20 - \tfrac{1}{4}x\right)^{\frac{1}{2}} + (3x)\left[\left(20 - \tfrac{1}{4}x\right)^{\frac{1}{2}}\right]'$$

$$= 3\left(20 - \tfrac{1}{4}x\right)^{\frac{1}{2}} + 3x\left(\tfrac{1}{2}\right)\left(20 - \tfrac{1}{4}x\right)^{-\frac{1}{2}}\left(20 - \tfrac{1}{4}x\right)'$$

$$= 3\left(20 - \tfrac{1}{4}x\right)^{\frac{1}{2}} + 3x\left(\tfrac{1}{2}\right)\left(20 - \tfrac{1}{4}x\right)^{-\frac{1}{2}}\left(-\tfrac{1}{4}\right)$$

$$= 3\sqrt{20 - \tfrac{1}{4}x} - \frac{3x}{8\sqrt{20 - \frac{1}{4}x}}$$

$$A'(5) = 3\sqrt{20 - \tfrac{1}{4}(5)} - \frac{3(5)}{8\sqrt{20 - \frac{1}{4}(5)}}$$

$$= 3\sqrt{20 - \tfrac{5}{4}} - \frac{15}{8\sqrt{20 - \frac{5}{4}}}$$

$$A'(5) \approx 12.56$$

After eating 5 lb of Sausage, George sensitivity is 12.56