Advanced Derivative Rules
4.3 – Differentiating Using Two Rules

Ex A: Find the derivative of each function. Factor answers when appropriate.

#1) $h(x) = (3x+2)^4(5x-7)^3$

$h'(x) = [(3x+2)^4]'(5x-7)^3 + (3x+2)^4[(5x-7)^3]'$

$h'(x) = 4(3x+2)^3(3x+2)'(5x-7)^3 + (3x+2)^4 3(5x-7)^2(5x-7)'$

$h'(x) = 4(3x+2)^3(3)(5x-7)^3 + (3x+2)^4 3(5x-7)^2(5)$

$h'(x) = 3(3x+2)^3(5x-7)^2[4(5x-7) + (3x+2)5]$

$h'(x) = 3(3x+2)^3(5x-7)^2[20x-28 + 15x+10]$

$h'(x) = 3(3x+2)^3(5x-7)^2[35x-18]$

#2) $y = \left(\frac{x^2}{x-1}\right)^5$

$y' = 5\left(\frac{x^2}{x-1}\right)^4 \cdot \left(\frac{x^2}{x-1}\right)'$

$y' = 5\frac{x^8}{(x-1)^4} \cdot \frac{(x^2)'(x-1) - x^2(x-1)'}{(x-1)^2}$

$y' = \frac{5x^8[2x(x-1) - x^2(1)]}{(x-1)^6}$

$y' = \frac{5x^8[2x^2-2x-x^2]}{(x-1)^6}$

$y' = \frac{5x^8(x^2-2x)}{(x-1)^6}$

$y' = \frac{5x^9(x-2)}{(x-1)^6}$

#3) $g(x) = [x^2 - (x^3-6)^4]^8$

$g'(x) = 8[x^2-(x^3-6)^4]^7 \cdot [x^2-(x^3-6)^4]'$

$g'(x) = 8[x^2-(x^3-6)^4]^7[2x - 4(x^3-6)^3 \cdot (x^3-6)']$

$g'(x) = 8[x^2-(x^3-6)^4]^7[2x - 4(x^3-6)^3 3x^2]$

$g'(x) = 8[x^2-(x^3-6)^4]^7[2x - 12x^2(x^3-6)^3]$

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#4) $h(x) = (x+1)^{10}(x-1)^8$

$h'(x) = [(x+1)^{10}]'(x-1)^8 + (x+1)^{10}[(x-1)^8]'$

$= 10(x+1)^9(x+1)'(x-1)^8 + (x+1)^{10} \cdot 8(x-1)^7 \cdot (x-1)'$

$= 10(x+1)^9(1)(x-1)^8 + (x+1)^{10} \cdot 8(x-1)^7(1)$

$= 2(x+1)^9(x-1)^7[5(1)(x-1) + 4(x+1)(1)]$

$= 2(x+1)^9(x-1)^7[5x-5+4x+4]$

$= 2(x+1)^9(x-1)^7(9x-1)$

#5) $y = \left(\frac{x+1}{5x^6}\right)^6$

$y' = 6\left(\frac{x+1}{5x^6}\right)^5 \cdot \left(\frac{x+1}{5x^6}\right)'$

$= \frac{6(x+1)^5}{(5x^6)^5} \cdot \frac{(x+1)'(5x^6) - (x+1)(5x^6)'}{(5x^6)^2}$

$= \frac{6(x+1)^5}{3125x^{30}} \cdot \frac{(1)(5x^6) - (x+1)(30x^5)}{25x^{12}}$

$= \frac{6(x+1)^5}{3125x^{30}} \cdot \frac{5x^6 - 30x^6 - 30x^5}{25x^{12}}$

$= \frac{6(x+1)^5(-25x^6 - 30x^5)}{78125x^{42}}$

$y' = \frac{6(x+1)^5(-5x^5)(5x+6)}{78125x^{42}}$

$y' = \frac{-6(x+1)^5(5x+6)}{15625x^{37}}$

#6) $g(x) = [5x^3 + (x^3+5)^8]^7$

$g'(x) = 7[5x^3 + (x^3+5)^8]^6 \cdot [5x^3 + (x^3+5)^8]'$

$= 7[5x^3 + (x^3+5)^8]^6[15x^2 + 8(x^3+5)^7 \cdot (x^3+5)']$

$g' = 7[5x^3 + (x^3+5)^8]^6[15x^2 + 8(x^3+5)^7 \cdot (3x^2)]$

#7) If $y = \tan^5(2x)$ find y'.

$y = [\tan(2x)]^5$

$y' = 5[\tan(2x)]^4 \cdot [\tan(2x)]'$

$y' = 5\tan^4(2x)\ \sec^2(2x) \cdot (2x)'$

$y' = 5\tan^4(2x)\ \sec^2(2x)\ (2)$

$y' = 10\tan^4(2x)\ \sec^2(2x)$

#8) If $y = -4\cot^3(7x)$ find y'.

$y = -4[\cot(7x)]^3$

$y' = -12[\cot(7x)]^2 \cdot [\cot(7x)]'$

$y' = -12\ \cot^2(7x) \cdot [-\csc^2(7x)] \cdot (7x)'$

$y' = -12\ \cot^2(7x) \cdot [-\csc^2(7x)] \cdot (7)$

$y' = 84\ \cot^2(7x) \cdot \csc^2(7x)$

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#9) If $y = \sec(\cos(4x))$ find y'.

$y' = \sec(\cos(4x))\tan(\cos(4x)) \cdot [\cos(4x)]'$

$y' = \sec(\cos(4x))\tan(\cos(4x))(-\sin(4x))(4x)'$

$y' = \sec(\cos(4x))\tan(\cos(4x))(-\sin(4x))\,4$

$y' = -4\sec(\cos(4x))\tan(\cos(4x))\sin(4x)$

Eraser much?

#10) If $y = \sin^2(x) + \sin(x) + \cos^2(x)$ find y'.

$y = \sin^2(x) + \cos^2(x) + \sin(x)$

$y = 1 + \sin(x)$

$y' = 0 + \cos(x)$

$y' = \cos(x)$