Advanced Derivative Rules
4.4 – Nondifferentiable Functions

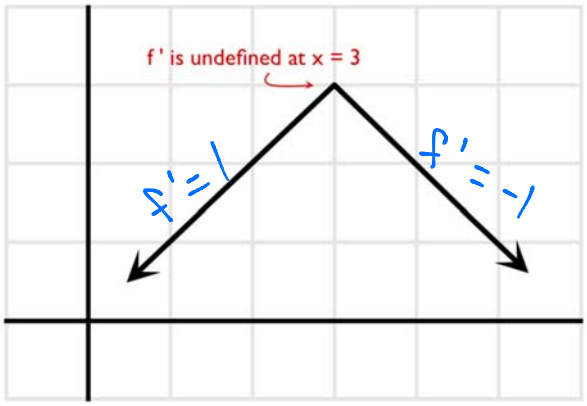
Thus far we have learned to find derivatives by the definition of derivative, the Power Rule, the Sum & Difference Rules, the Product Rule, the Quotient Rule, and the Chain Rule. However, there are functions that cannot be differentiated at certain values. These are called nondifferentiable functions. Knowing where a function is not differentiable is the focus of this section.

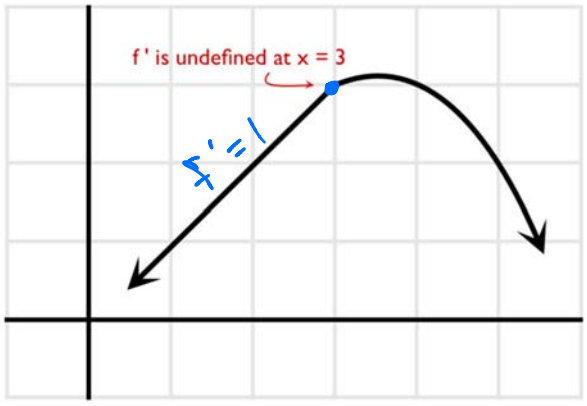
Summary of Nondifferentiable Functions

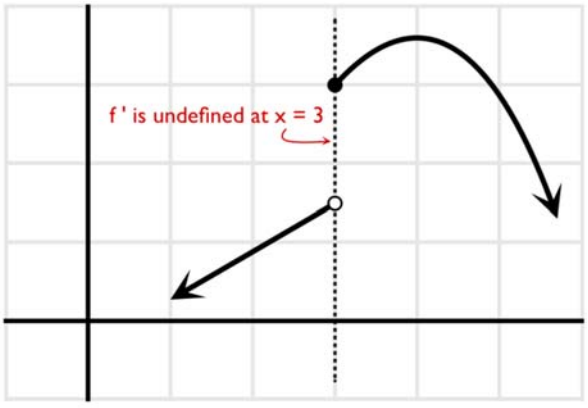
Therefore, if a function f satisfies any of the following conditions:

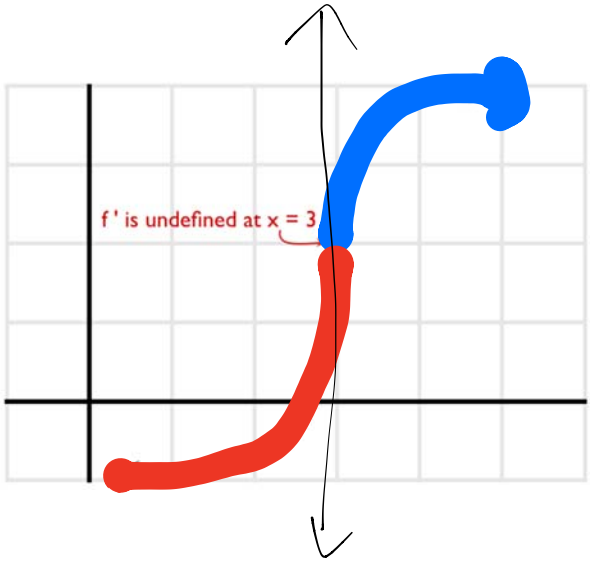
1. f has a corner point at $x = c$,
2. f has a vertical tangent at $x = c$,
3. f is discontinuous at $x = c$,

then f will not be differentiable at c.









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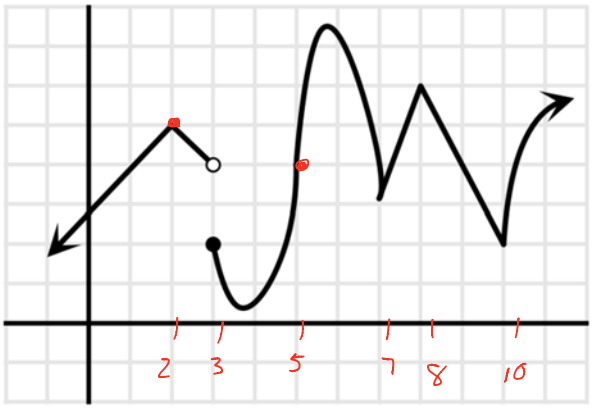
4.4 – Nondifferentiable Functions

Ex A: Find the x-values at which the derivative is undefined.



Ex B: Graph $f(x) = |x|$ and show it's not differentiable at $x = 0$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{|x+h| - |x|}{h}$$

$$f'(0) = \lim_{h\to 0} \frac{|0+h| - |0|}{h}$$

$$f'(0) = \lim_{h\to 0} \frac{|h|}{h}$$

$$= \lim_{h\to 0^-} \frac{|h|}{h}$$

$$= \lim_{h\to 0^-} \frac{|-h|}{-h}$$

$$= \lim_{h\to 0^-} \frac{+h}{-h}$$

$$= \lim_{h\to 0^-} -1$$

$$= -1$$

$$= \lim_{h\to 0^+} \frac{|h|}{h}$$

$$= \lim_{h\to 0^+} \frac{|+h|}{+h}$$

$$= \lim_{h\to 0^+} \frac{+h}{+h}$$

$$= \lim_{h\to 0^+} +1$$

$$= 1$$