Advanced Derivative Rules
Review Chapter 4

Find the derivative of each function.

#1) $g(x) = (5x^4 + 6x)^2$

$g'(x) = 2(5x^4+6x)(5x^4+6x)'$

$= 2(5x^4+6x)(20x^3+6)$

#2) $k(x) = \left(\frac{1}{2}x^4 + 3x^2 + 10\right)^{10}$

$k'(x) = 10\left(\frac{1}{2}x^4+3x^2+10\right)^9\left(\frac{1}{2}x^4+3x^2+10\right)'$

$= 10\left(\frac{1}{2}x^4+3x^2+10\right)^9(2x^3+6x)$

#3) $f(x) = \sqrt[5]{(3x+1)^8} = (3x+1)^{8/5}$

$f'(x) = \frac{8}{5}(3x+1)^{3/5}(3x+1)'$

$= \frac{8}{5}\sqrt[5]{(3x+1)^3}\,(3)$

$= \frac{24}{5}\sqrt[5]{(3x+1)^3}$

#4) $y = \sqrt{7x-1} = (7x-1)^{\frac{1}{2}}$

$y' = \frac{1}{2}(7x-1)^{-\frac{1}{2}}(7x-1)'$

$= \frac{1}{2\sqrt{7x-1}}(7)$

$y' = \frac{7}{2\sqrt{7x-1}}$

#5) $y = \left(\frac{5}{2x^2+5}\right)^5$

$y' = 5\left(\frac{5}{2x^2+5}\right)^4\left(\frac{5}{2x^2+5}\right)'$

$= 5\left(\frac{5}{2x^2+5}\right)^4 \cdot \frac{5'(2x^2+5) - 5(2x^2+5)'}{(2x^2+5)^2}$

$= 5\,\frac{5^4}{(2x^4+5)^4} \cdot \frac{0(2x^2+5) - 5(4x)}{(2x^2+5)^2}$

$= \frac{5^5(0 - 20x)}{(2x^4+5)^6}$

$= \frac{3125(-20x)}{(2x^4+5)^6}$

$y' = \frac{-62,500x}{(2x^4+5)^6}$

#6) $y = \frac{1}{\sqrt{3x-10}} = (3x-10)^{-\frac{1}{2}}$

$y' = -\frac{1}{2}(3x-10)^{-3/2}(3x-10)'$

$= \frac{-1}{2\sqrt{(3x-10)^3}}(3)$

$= \frac{-3}{2\sqrt{(3x-10)^3}}$

#7) $\frac{d}{dx}[\cos^5(x)]$

$= 5\cos^4(x) \cdot \cos'(x)$

$= 5\cos^4(x)\left(-\sin(x)\right)$

$= -5\cos^4(x)\sin(x)$

#8) $\frac{d}{dx}[\tan^2(x)]$

$= 2\tan(x) \cdot \tan'(x)$

$= 2\tan(x)\sec^2(x)$

#9) $\frac{d}{dx}[\csc^{16}(x)]$

$= 16\csc^{15}(x) \cdot \csc'(x)$

$= 16\csc^{15}(x)\left(-\csc(x)\cot(x)\right)$

$= -16\csc^{16}(x)\cot(x)$

#10) $\frac{d}{dx}[\sin(x^3)]$

$= \cos(x^3) \cdot (x^3)'$

$= 3x^2\cos(x^3)$

#11) $\frac{d}{dx}[\cot(8x)]$

$= -\csc^2(8x) \cdot (8x)'$

$= -8\csc^2(8x)$

#12) $\frac{d}{dx}[\sec(x^{10})]$

$= \sec(x^{10})\tan(x^{10}) \cdot (x^{10})'$

$= 10x^9\sec(x^{10})\tan(x^{10}).$

#13) $\frac{d}{dx}(5x^3 \sin(x))$

$= (5x^3)' \sin(x) + 5x^3 \cdot \sin'(x)$

$= 15x^2 \sin(x) + 5x^3 \cos(x)$

#14) If $f(x) = 4x^2 \cos(x)$, find $f'(x)$.

$f'(x) = (4x^2)' \cos(x) + 4x^2 \cdot \cos'(x)$

$= 8x \cdot \cos(x) + 4x^2(-\sin(x))$

$= 8x \cos(x) - 4x^2 \sin(x)$

#15) $\frac{d}{dx}[x \tan(x)]$

$= x' \cdot \tan(x) + x \cdot \tan'(x)$

$= 1 \cdot \tan(x) + x \cdot \sec^2(x)$

$= \tan(x) + x \sec^2(x)$

#16) $\frac{d}{dx}[\tan(x)\cot(x)] = \frac{d}{dx}[1]$

$=$ ☺

#17) $\frac{d}{dx}\left(\frac{\sin(x)}{7x-5}\right) = \frac{\sin'(x) \cdot (7x-5) - \sin(x)(7x-5)'}{(7x-5)^2}$

$= \frac{\cos(x)(7x-5) - \sin(x) \cdot (7)}{(7x-5)^2}$

$= \frac{(7x-5)\cos(x) - 7\sin(x)}{(7x-5)^2}$

Find the derivative of each function. Factor answers when appropriate.

#18) $h(x) = (x^2+2)^4(2x-1)^5$

$$h'(x) = \left[(x^2+2)^4\right]'(2x-1)^5 + (x^2+2)^4\left[(2x-1)^5\right]'$$

$$= 4(x^2+2)^3(x^2+2)'(2x-1)^5 + (x^2+2)^4\, 5(2x-1)^4(2x-1)'$$

$$= 4(x^2+2)^3(2x)(2x-1)^5 + (x^2+2)^4\, 5(2x-1)^4(2)$$

$$= (x^2+2)^3(2x-1)^4(2)\left[4(x)(2x-1) + (x^2+2)(5)\right]$$

$$= 2(x^2+2)^3(2x-1)^4\left[8x^2-4x+5x^2+10\right]$$

$$h'(x) = 2(x^2+2)^3(2x-1)^4\left[13x^2-4x+10\right]$$

#19) $y = \left(\frac{x+1}{x-1}\right)^2$

$$y' = 2\left(\frac{x+1}{x-1}\right)^1\left(\frac{x+1}{x-1}\right)'$$

$$= 2\,\frac{x+1}{x-1}\cdot\frac{(x+1)'(x-1)-(x+1)(x-1)'}{(x-1)^2}$$

$$= 2\cdot\frac{x+1}{x-1}\;\frac{1(x-1)-(x+1)(1)}{(x-1)^2}$$

$$= 2\,\frac{(x+1)\cdot[x-1-x-1]}{(x-1)^3}$$

$$= \frac{2(x+1)(-2)}{(x-1)^3}$$

$$y' = \frac{-4(x+1)}{(x-1)^3}$$

#20) $y = \sqrt{3+\sqrt{x}}$

$$y' = \frac{1}{2}(3+\sqrt{x})^{-\frac{1}{2}}\left(3+x^{\frac{1}{2}}\right)'$$

$$= \frac{1}{2\sqrt{3+\sqrt{x}}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= \frac{1}{2\sqrt{3+\sqrt{x}}}\cdot\frac{1}{2\sqrt{x}}$$

$$\sqrt{3+\sqrt{x}}\cdot\sqrt{x}$$

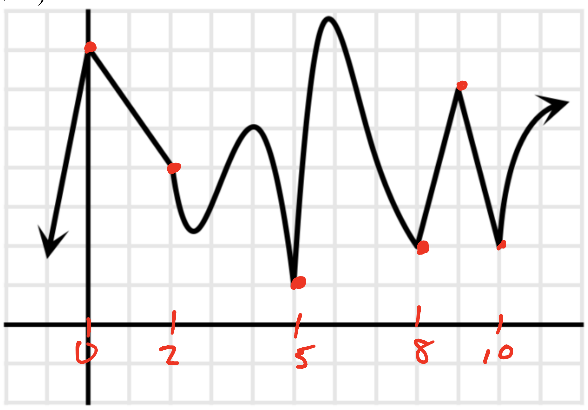
$$\sqrt{(3+\sqrt{x})x}$$

$$\sqrt{3x+x\sqrt{x}}$$

$$y' = \frac{1}{4\sqrt{3x+x\sqrt{x}}}$$

Find the values of x where the function has no derivative.

#21)



$x = 0, 2, 5, 8, 9, 10$

George is his own grandpa

#22) While playing with his favorite animals (the allusive dust bunnies) in his attic, George stumbles upon an old photo album. As he admires how attractive his grandma was pre-World War I, George decides to go back in time to try and put the moves on the (previously young) lady. Presuming it will take a lot of money to make a time machine, George decides to make some quick cash by making and selling time machines. He deduces a cost function of $C(x) = \sqrt{200x^2 + 1}$ dollars, where x is the number of time machines made.

a. Find the marginal cost function

b. Evaluate $MC(x)$ when 5 time machines have been produced and interpret your answer.

a.

$$MC(x) = \frac{1}{2}(200x^2+1)^{-\frac{1}{2}}(200x^2+1)'$$

$$= \frac{1}{2\sqrt{200x^2+1}}(400x)$$

$$= \frac{400x}{2\sqrt{200x^2+1}}$$

$$MC(x) = \frac{200x}{\sqrt{200x^2+1}}$$

b. $MC(5) = \dfrac{200(5)}{\sqrt{200(5)^2+1}}$

$$= \frac{1{,}000}{\sqrt{200(25)+1}}$$

$$= \frac{1{,}000}{\sqrt{5000+1}}$$

$$= \frac{1{,}000}{\sqrt{5001}}$$

$MC(5) \approx \$14.14$/time machine

when 5 time machines have been produced, it will cost \$14.14 to make the next one.

Mutual Funds

#23) George's business venture, We Got Time, didn't pan out as he had expected because he ran out of time. George then decides to try his luck at investing. While digging through his couch cushions, he finds a \$3. He plans to invest into a mutual fund paying r% in returns annually. In 2 years the value of the mutual fund will be $V(r) = 3(1 + 0.01r)^2$ dollars.

a. Find V(12) and interpret your answer.
b. Find $V'(12)$ and interpret your answer.

r = rate of return
V = value in \$

a. $V(12) = 3[1+.01(12)]^2$
$= 3(1+(.12))^2$
$= 3(1.12)^2$
$= 3(1.2544)$
$V(12) = \$3.76$

At 12% return, the investment will be \$3.76 in 2 years.

b. $V'(r) = 2\cdot 3(1+0.01r)(1+0.01r)'$
$= 6(1+0.01r)(.01)$
$V'(r) = .06 + .0006r$
$V'(12) = .06 + .0006(12)$
$= .06 + .0072$
$= \$.0672/\%$ return

At 12% return & 2 years, the investment is increasing by about 6.72 ¢ per percent rate of return.

Tusky's Population

#24) George's investment did not pan out as he had planned. With a rotten stench of failure hanging in the air, George decided to bath. The population of bacteria in the tub x minutes after George starts his bath is expected to be $P(x) = \sqrt{x^2 + 5}$ thousand for $0 \le x \le 5$.

a. Find $P(2)$ and interpret your answer.
b. Find $P'(2)$ and interpret your answer.

X = minutes in tub
P = bacteria in thousands

a. $P(2) = \sqrt{(2)^2+5}$
$= \sqrt{4+5}$
$= \sqrt{9}$
$P(2) = 3$ thousand bacteria

After being in the tub for 2 minutes, there are 3000 bacteria in the tub.

b. $P(x) = (x^2+5)^{\frac{1}{2}}$
$P'(x) = \frac{1}{2}(x^2+5)^{-\frac{1}{2}}(x^2+5)'$
$= \frac{1}{2\sqrt{x^2+5}}(2x)$
$P'(x) = \frac{x}{\sqrt{x^2+5}}$

$P'(2) = \frac{2}{\sqrt{(2)^2+5}}$
$= \frac{2}{\sqrt{4+5}}$
$= \frac{2}{\sqrt{9}}$
$P'(2) = \frac{2}{3}$ $\frac{\text{thousand bact}}{\text{second}}$.

After being in the tub for 2 minutes, the bacteria in the tub is increasing by 666 bacteria per second.