#19) $f(x) = 0.01x^5 - 0.05x$

① CV

$f'(x) = 0.05x^4 - 0.05$

$0 = 0.05(x^4 - 1)$

$0 = 0.05(x^2-1)(x^2+1)$

$0 = 0.05(x-1)(x+1)(x^2+1)$

ZOH

$0 = 0.05(x-1)(x+1)(x^2+1)$

$0 \neq 0.05$ $\quad$ $0 = x-1$, $1 = x$ $\quad$ $0 = x+1$, $-1 = x$ $\quad$ $0 = x^2+1$, $-1 = x^2$, $\pm\sqrt{-1} = x$ imaginary

CV: $x = -1, 1$

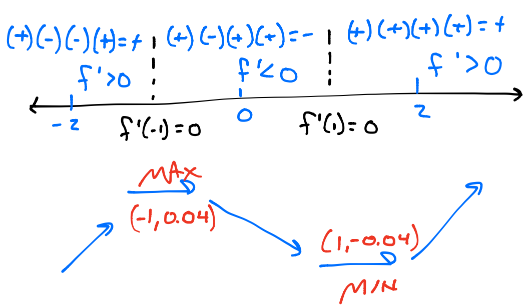
② CP

$f(-1) = 0.04$

$f(1) = -0.04$

CP $(-1, 0.04)$ $(1, -0.04)$

③ $f'(x) = 0.05(x-1)(x+1)(x^2+1)$



⑦ y-int

$f(0) = 0$

④ CV

$f''(x) = 0.20x^3$

$0 = 0.20x^3$

ZOH

$0 = 0.20x^3$

$0 = x^3$

$0 = x$

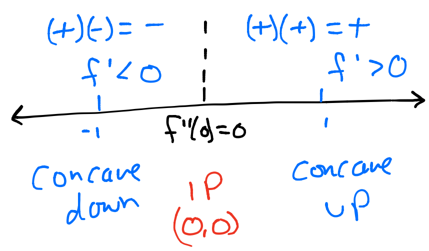
CV: $x = 0$

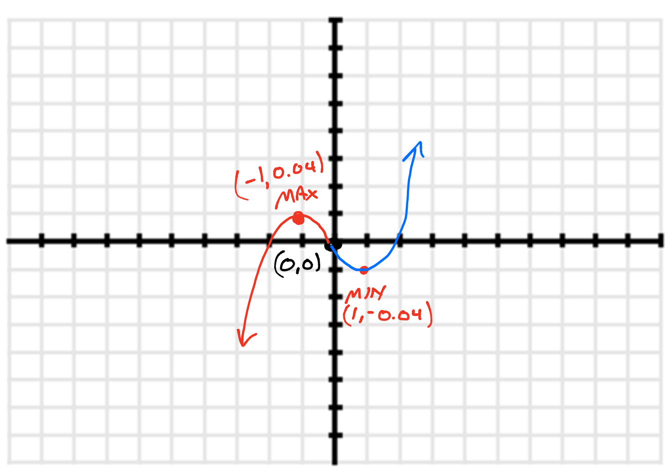
⑤ CP

$f(0) = 0$

CP(0, 0)

⑥ $f''(x) = 0.20x^3$





#20) $f(x) = x^3 - 2x^2 + x + 11$

① CV

$f'(x) = 3x^2 - 4x + 1$

$0 = (3x^2 - 3x) + (-x + 1)$

$0 = 3x(x-1) + -1(x-1)$

$0 = (x-1)(3x-1)$

ZON

$0 = x - 1$, $1 = x$; $3x - 1 = 0$, $3x = 1$, $x = \frac{1}{3}$

CV: $x = \frac{1}{3}, 1$

② CP

$f(\frac{1}{3}) = 11.15$

$f(1) = 11$

CP: $(\frac{1}{3}, 11.15), (1, 11)$

③

$f'(x) = (x-1)(3x-1)$

$(-)(-) = +$ | $(-)(+) = -$ | $(+)(+) = +$

$f' > 0$ | $f' < 0$ | $f' > 0$

0 $f'(\frac{1}{3}) = 0$ $.9$ $f'(1) = 0$ 2

MAX $(\frac{1}{3}, 11.15)$

MIN $(1, 11)$

④ CV

$f''(x) = 6x - 4$

$0 = 2(3x - 2)$

ZON

$0 \neq 2$; $3x - 2 = 0$, $3x = 2$, $x = \frac{2}{3}$

CV: $x = \frac{2}{3}$

⑤ CP

$f(\frac{2}{3}) = 11.07$

CP: $(\frac{2}{3}, 11.07)$

⑥

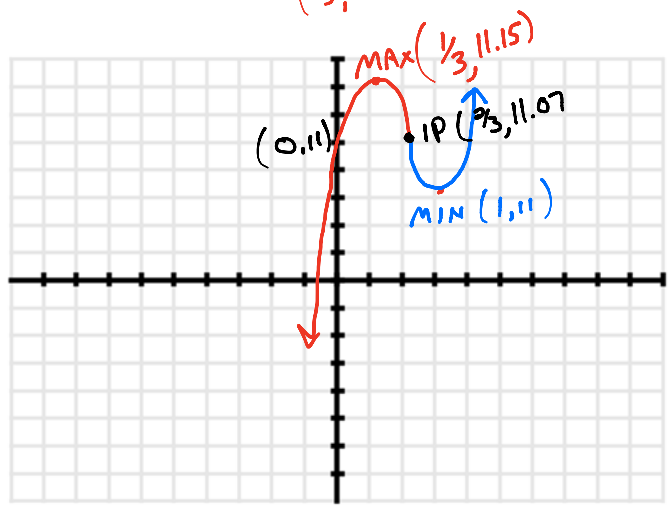
$f''(x) = 2(3x - 2)$

$(+)(-) = -$ | $(+)(+) = +$

$f'' < 0$ | $f'' > 0$

0 $f''(\frac{2}{3}) = 0$ 1

Concave down IP $(\frac{2}{3}, 11.07)$ Concave up



⑦ Y-int

$\boxed{f(0) = 11}$

Domain Restriction!

Graphing & Basic Optimization
5.1A – Graphing Using Derivatives

$\left[(400-x^2)^{\frac{1}{2}}\right]'$

$= \frac{1}{2}(400-x^2)^{-\frac{1}{2}}(400-x^2)'$

$= \frac{1}{2}(400-x^2)^{-\frac{1}{2}}(-2x)$

#21) $f(x) = \sqrt{400 - x^2}$

① CV

$f'(x) = \frac{1}{2}(400-x^2)^{-\frac{1}{2}}(-2x)$

$0 = \frac{-x}{\sqrt{400-x^2}}$

ZON

$0 = -x$

$0 = x$

CV: $x = 0$

ZOD

$0 = \sqrt{400-x^2}$

$0 = 400 - x^2$

$x^2 = 400$

$x = \pm 20$

CV: $x = -20, 20$

② CP

$f(-20) = 0$

$f(0) = 20$

$f(20) = 0$

CP: $(-20,0), (0,20), (20,0)$

③ $f'(x) = \frac{-x}{\sqrt{400-x^2}}$

$\frac{(+)}{\sqrt{-}}$ = und | $\frac{(+)}{\sqrt{+}} = +$ | $\frac{(-)}{\sqrt{+}} = -$ | $\frac{(-)}{\sqrt{-}}$ = und

f' = und | $f' > 0$ | $f' < 0$ | f' = und

-30 $\quad f'(-20)$ = und $\quad -1$ $\quad f'(0) = 0$ $\quad 1$ $\quad f'(20)$ = und $\quad 30$

und $\quad$ MIN $(-20,0)$ $\quad$ MAX $(0,20)$ $\quad$ MIN $(20,0)$ $\quad$ und

④ CV

$f''(x) = \frac{(-x)' \cdot \sqrt{400-x^2} - (-x)\left(\sqrt{400-x^2}\right)'}{\left(\sqrt{400-x^2}\right)^2}$

Chain Rule

$0 = \frac{-1 \cdot \sqrt{400-x^2} + x\left(\frac{1}{2}\right)(400-x^2)^{-\frac{1}{2}}(-2x)}{400-x^2}$

$= \frac{(400-x^2)^{-\frac{1}{2}}\left[-1(400-x^2) - x^2\right]}{(400-x^2)^1}$

$= (400-x^2)^{-\frac{3}{2}}(-400 + x^2 - x^2)$

$0 = \frac{-400}{\left(\sqrt{400-x^2}\right)^3}$

ZON

$0 \neq -400$

ZOD

$0 = \left(\sqrt{400-x^2}\right)^3$

$0 = 400 - x^2$

$x^2 = 400$

CV: $x = \pm 20$

⑤ CP

$f(-20) = 0$

$f(20) = 0$

CP: $(-20,0), (20,0)$

⑥ $f''(x) = \frac{-400}{\left(\sqrt{400-x^2}\right)^3}$

$\frac{-}{\sqrt{-}}$ = und | $\frac{-}{\sqrt{+}} = -$ | $\frac{-}{\sqrt{-}}$ = und

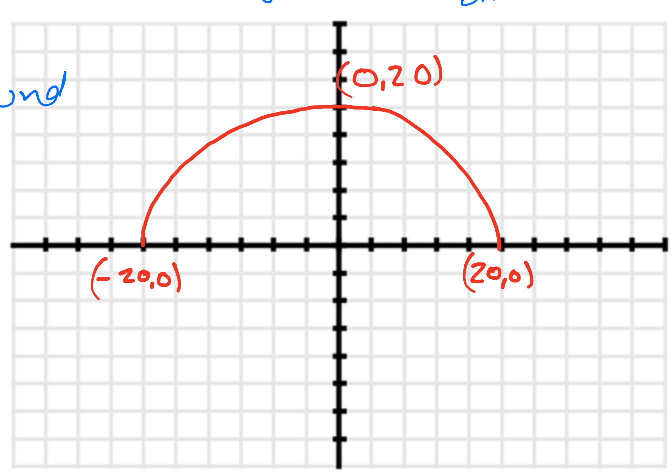
f'' = und | $f'' < 0$ | f'' = und

-5 mill $\quad f''(-20)$ = und $\quad 0 \quad f''(20)$ = und $\quad 5$ mill

und $\quad$ Concave DN $\quad$ und

⑦ y-int

$\boxed{f(0) = 20}$



#22) $f(x) = \frac{1}{x^2-2x-8}$ RATIONAL!

$f(x) = \frac{1}{(x-4)(x+2)}$

Holes & Asymptotes

VA
$(x-4)(x+2)=0$
$x-4=0$, $x+2=0$
$x=4$, $x=-2$

HA
$n ? d$
$0 < 2$, so HA $y=0$

① CV

$f(x) = (x^2-2x-8)^{-1}$

$f'(x) = -(x^2-2x-8)^{-2}(2x-2)$

$0 = \frac{-(2x-2)}{(x^2-2x-8)^2}$

$0 = \frac{-2(x-1)}{[(x-4)(x+2)]^2}$

ZON
$0 = -2(x-1)$
$0 \neq -2$, $0 = x-1$, $1 = x$
CV: $x=1$

ZOD
$0 = [(x-4)(x+2)]^2$
$0 = (x-4)(x+2)$
$0 = x-4$, $4 = x$; $0 = x+2$, $-2 = x$
CV: $x = -2, 4$

② $f(-2) = $ und
$f(1) = -\frac{1}{9}$
$f(4) = $ und
CP $(1, -\frac{1}{9})$

③ $f'(x) = \frac{-2(x-1)}{[(x-4)(x+2)]^2}$

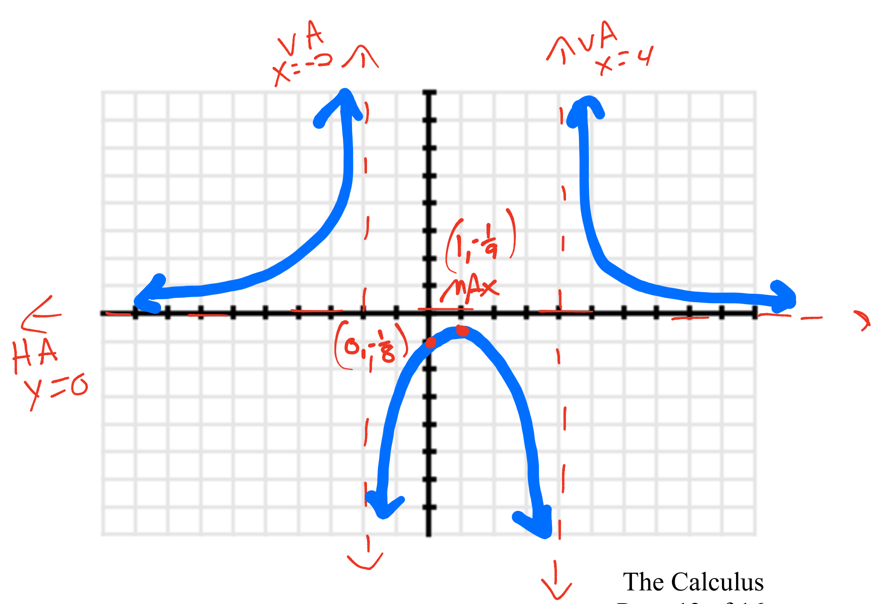
$\frac{(-)(-)}{+} = +$ | $\frac{(-)(-)}{+} = +$ | $\frac{(-)(+)}{+} = -$ | $\frac{(-)(+)}{+} = -$

$f' > 0$ | $f' > 0$ | $f' < 0$ | $f' < 0$

-3 | $f'(-2) = $ und | 0 | $f'(1) = 0$ | 2 | $f'(4) = $ und | 5

VA | MAX $(1, -\frac{1}{9})$ | VA

⑦ Y-int
$f(0) = -\frac{1}{8}$



#23) $f(x) = \frac{8}{x^2+4}$ RATIONAL!

$f(x) = \frac{8}{x^2+4}$

HOLES & Asymptotes

VA
$x^2+4=0$
$x^2=-4$
$x=\pm\sqrt{-4}$
$x=$ imagn
NO VA

HA
$n?d$
$0<2$, so HA $y=0$

CV

① $f(x) = 8(x^2+4)^{-1}$

$f'(x) = -8(x^2+4)^{-2}(x^2+4)'$

$0 = -8(x^2+4)^{-2}(2x)$

$0 = \frac{-16x}{(x^2+4)^2}$

ZON
$0=-16x$
$0=x$
CV: $x=0$

ZOD
$0=(x^2+4)^2$
$0=x^2+4$
$-4=x^2$
$\pm\sqrt{-4}=x$
imaginary
NO CV

② CP
$f(0)=2$
CP: $(0,2)$

③ $f' = \frac{-16x}{(x^2+4)^2}$

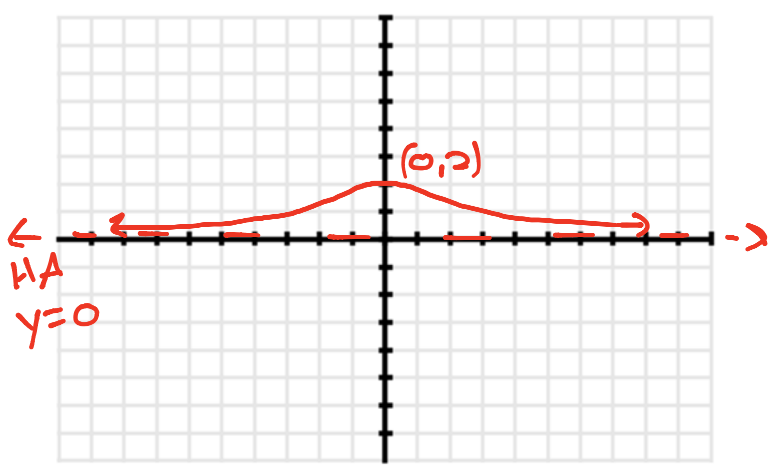
$\frac{(-)(-)}{+} = +$ | $\frac{(-)(+)}{+} = -$

$f'>0$ | $f'<0$

-1 | $f'(0)=0$ | 1

MAX
$(0,2)$

⑦ y-int
$f(0)=2$



#24) $f(x) = \frac{x^2}{x^2+1}$ RATIONAL!

No Holes

VA

$x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1}$

$x=$ dne.

HA

$n ? d$

$2=2$, so HA $y=1$

① CV

$$f'(x) = \frac{(x^2)'(x^2+1) - x^2(x^2+1)'}{(x^2+1)^2}$$

$$= \frac{2x(x^2+1) - x^2(2x)}{(x^2+1)^2}$$

$$= \frac{2x^3 + 2x - 2x^3}{(x^2+1)^2}$$

$$0 = \frac{2x}{(x^2+1)^2}$$

ZON

$0 = 2x$

$0 = x$

CV: $x=0$

ZOD

$(x^2+1)^2 = 0$

$x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1}$

$x=$ d.n.e.

② CP

$f(0)=0$

CP $(0,0)$

③ $f'(x) = \frac{2x}{(x^2+1)^2}$

$\frac{(-)}{(+)} = -$

$\frac{+}{+} = +$

$f'<0$

$f'>0$

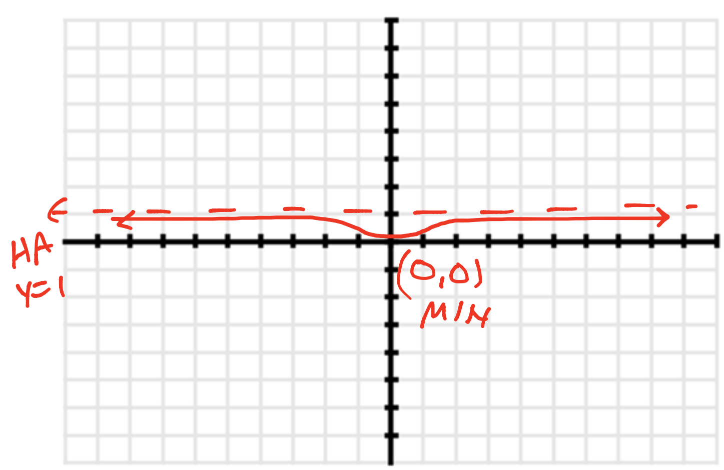
-1

$f'(0)=0$

1

$(0,0)$

MIN



#25) $f(x) = \frac{x^2}{x-3}$ RATIONAL!

No Holes

VA
$x-3=0$
$x=3$

HA
$n ? d$
$2>1$, so no HA

SA
$n \overset{?}{=} d+1$
$2=1+1$, so SA $y=x+3$

$$x-3\overline{)x^2}\quad x+3 \;\; R\,9$$
$$+(-x^2+3x)$$
$$3x$$
$$+(-3x+9)$$
$$9$$

① CV

$$f'(x)=\frac{(x^2)'(x-3)-x^2(x-3)'}{(x-3)^2}$$
$$=\frac{2x(x-3)-x^2(1)}{(x-3)^2}$$
$$=\frac{2x^2-6x-x^2}{(x-3)^2}$$
$$=\frac{x^2-6x}{(x-3)^2}$$
$$0=\frac{x(x-6)}{(x-3)^2}$$

ZON
$0=x(x-6)$
$0=x$, $0=x-6$, $6=x$
CV: $x=0,6$

ZOD
$0=(x-3)^2$
$0=x-3$
$3=x$
CV: $x=3$

② CP
$f(0)=0$
$f(3)=$ und
$f(6)=12$
CP $(0,0)$, $(6,12)$

③ $f'(x)=\frac{x(x-6)}{(x-3)^2}$

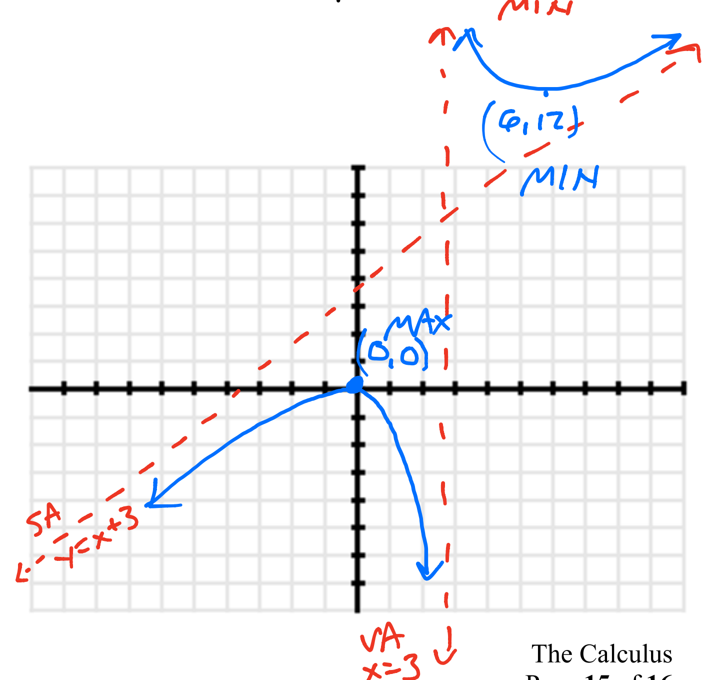
$\frac{(-)(-)}{+}=+$ | $\frac{(+)(-)}{+}=-$ | $\frac{(+)(-)}{+}=-$ | $\frac{(+)(+)}{+}=+$

$f'>0$ | $f'<0$ | $f'<0$ | $f'>0$

-1 | $f'(0)=0$ | 1 | $f'(3)=$ und | 4 | $f'(6)=0$ | 7

$(0,0)$ MAX

$(6,12)$ MIN



⑦ Y-int
$f(0)=0$

Answers

#1)	positive $(-\infty, -2)u(0, \infty)$ negative $(-2, 0)$	
#2)	positive $(0, 4)$,	negative $(-\infty, 0)u(4, \infty)$
#3)	c	
#4)	a	
#5)	d	
#6)	b	
#7)	1st derivative cv: -4, 4	2nd derivative cv:
#8)	1st derivative cv: -1, 5	2nd derivative cv:
#9)	1st derivative cv: -4, 0, 1	2nd derivative cv:
#10)	1st derivative cv: 3	2nd derivative cv:
#11)	1st derivative cv: none	2nd derivative cv:
#12)	1st derivative cv: -1, $\frac{1}{3}$	2nd derivative cv:

#13) - #25) Use calculator to check your graphs.