Graphing & Basic Optimization
5.1 – Graphing Using Derivatives

We learned previously that the derivative of a function gives the _____ of the graph.

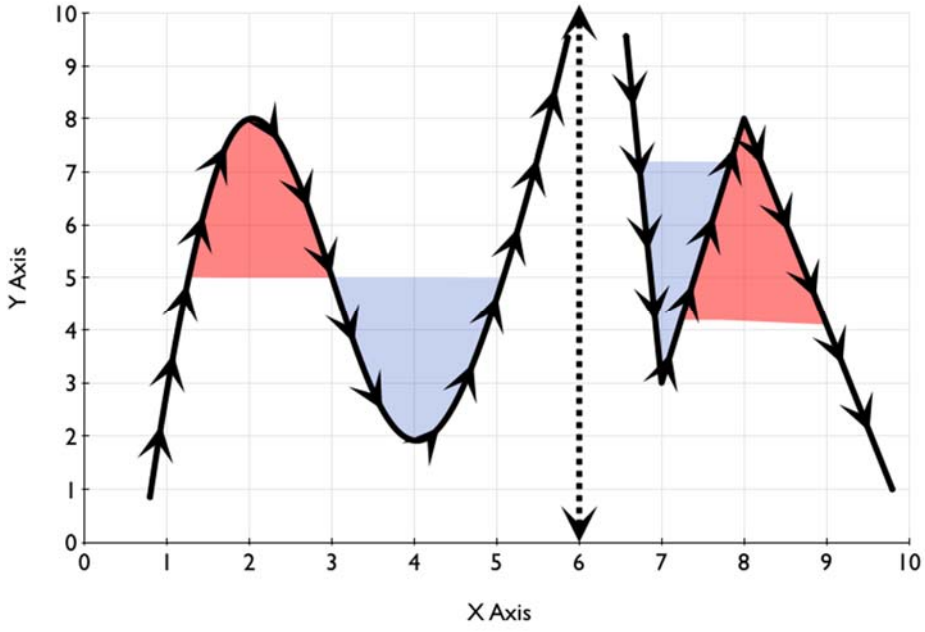
Critical Value: an x-value that changes the nature of a curve. A critical value of function g is an x-value in the domain of g that satisfies one of the following:

$g'(x) = 0$	$g''(x) = 0$
$g'(x)$ is undefined	$g''(x)$ is undefined

A critical value can produce a minimum point, a maximum point, an inflection point, or a vertical asymptote.

Critical Point: a point on a graph that changes the nature of the graph.

Relative Maximum Point: a point that is at least as *high* as the points relative to it on the curve on either side.
Relative Minimum Point: a point that is at least as *low* as the points relative to it on the curve on either side.
Inflection Point: a point on a graph that changes the concavity.



A minimum changes the graph from decreasing to increasing.

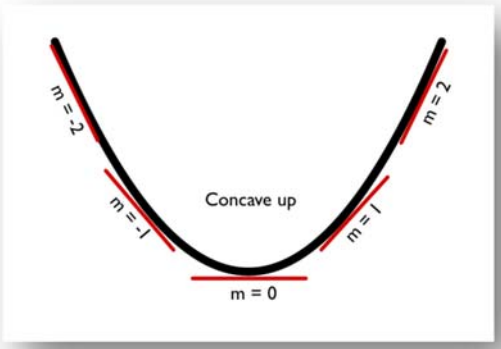
A maximum change the graph from increasing to decreasing.

An inflection point changes the concavity of the graph.

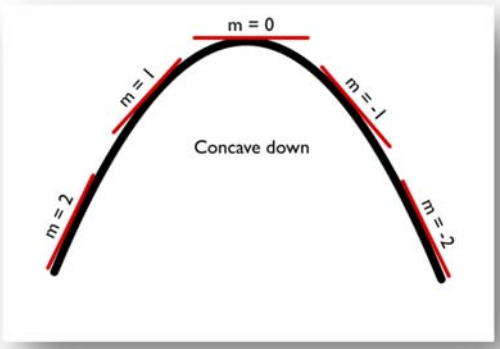
A vertical asymptote can change concavity and can change whether its increasing or decreasing.

Graphing via Derivatives

Step 1: Find CVs by finding when $f' = 0$ (ZON) and $f' = und$ (ZOD)
Step 2: Find CPs by evaluating $f(CV)$
Step 3: Make sign diagram for f'
Step 4: Find CVs by finding when $f'' = 0$ (ZON) and $f'' = und$(ZOD)
Step 5: Find CPs by evaluating $f(CV)$
Step 6: Make sign diagram for f''
Step 7: Find the y-intercept



Interpretation of graph:



Interpretation of graph:

Slope and Concavity

Distinguish carefully between slope and concavity: Slope measures steepness; concavity measures curl.

$f' > 0$ is increasing	$f'' > 0$ is concave up
$f' < 0$ is decreasing	$f'' < 0$ is concave down

Ex A: Draw part of a curve described below.

#1) Increasing and concave up

#2) Increasing and concave down

#3) Decreasing and concave up

#4) Decreasing and concave down

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Ex B: Guided Example (For day 1, do steps 1 – 3 and 7. For day 2, do steps 4 – 6.)
#1) A company's annual profit after *x* years is $f(x) = x^3 - 12x^2 - 60x + 15$ million dollars (for $x \geq 0$). Graph this function, show all relative extreme points and inflection points. Interpret the inflection point.

Step 1: Find CVs by finding when $f' = 0$ (ZON) and $f' = und$ (ZOD)

Step 2: Find CPs by evaluating $f(CV)$

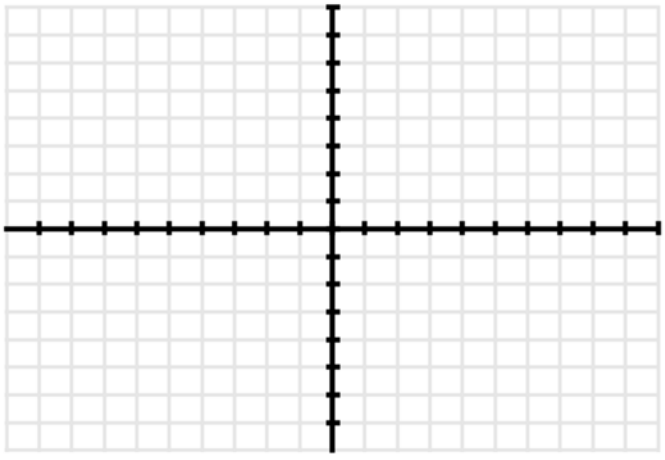
Step 3: Make sign diagram for f'

Step 4: Find CVs by finding when $f'' = 0$ (ZON) and $f'' = und$(ZOD)

Step 5: Find CPs by evaluating $f(CV)$

Step 6: Make sign diagram for f''

Step 7: Find the y-intercept



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5.1 – Graphing Using Derivatives

(For day 1, do steps 1 – 3 and 7. For day 2, do steps 4 – 6.)
#2) $f(x) = -x^4 + 4x^3 - 5$

Step 1:

Step 2:

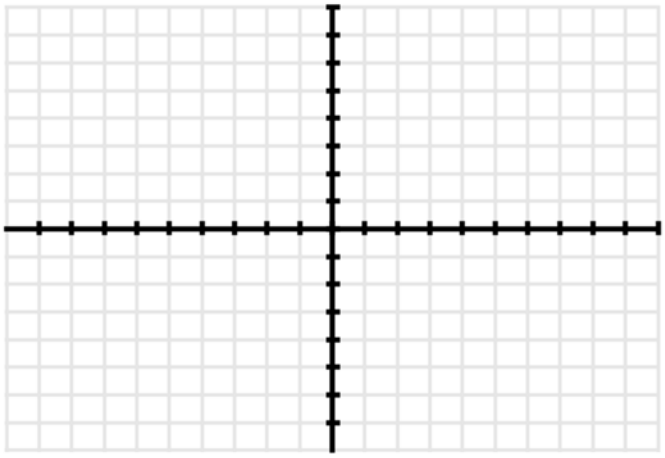
Step 3:

Step 4:

Step 5:

Step 6:

Step 7:



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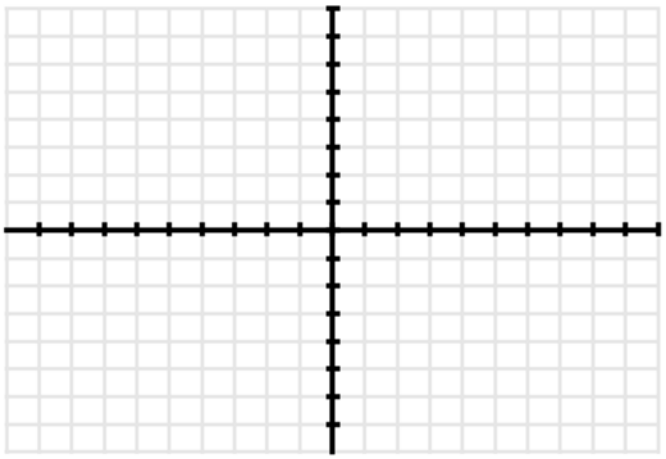
(For day 1, do steps 1 – 3 and 7. For day 2, do steps 4 – 6.)

#3) $f(x) = 9\sqrt[3]{(x-1)^2}$



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(For day 1, do steps 1 – 3 and 7. For day 2, do steps 4 – 6.)

#4) $f(x) = \frac{2}{x^2-4x}$