Graph the following function by hand by making a first derivative sign diagram and then a second derivative sign diagram.

#1) $f(x) = x^2(x^2 - 4)$

$f(x) = x^4 - 4x^2$

CV

$f'(x) = 4x^3 - 8x$

$0 = 4x(x^2 - 2)$

ZON

$0 = 4x$ | $0 = x^2 - 2$

$0 = x$ | $2 = x^2$

$\pm\sqrt{2} = x$

CV: $x = -\sqrt{2}, 0, \sqrt{2}$

CV

$f''(x) = 12x^2 - 8$

$0 = 12x^2 - 8$

$0 = 4(3x^2 - 2)$

ZON

$0 = 3x^2 - 2$

$2 = 3x^2$

$\frac{2}{3} = x^2$

CV: $\pm\sqrt{\frac{2}{3}} = x$

CP

$f(-\sqrt{2}) = -4$

$f(0) = 0$

$f(\sqrt{2}) = -4$

CP: $(-\sqrt{2}, -4), (0,0), (\sqrt{2}, -4)$

CP

$f(-\sqrt{2/3}) = -2.2$

$f(\sqrt{2/3}) = -2.2$

CP $(-\sqrt{\frac{2}{3}}, -2.2), (\sqrt{2/3}, -2.2)$

$f'(x) = 4x(x^2 - 2)$

$(+)(-)(+) = -$ | $(+)(-)(-) = +$ | $(+)(+)(-) = -$ | $(+)(+)(+) = +$

$f' < 0$ | $f' > 0$ | $f' < 0$ | $f' > 0$

-3 $f'(-\sqrt{2}) = 0$ -1 $f'(0) = 0$ 1 $f'(\sqrt{2}) = 0$ 3

$(-\sqrt{2}, -4)$ MIN; MAX $(0,0)$; $(\sqrt{2}, -4)$ MIN

$f''(x) = 4(3x^2 - 2)$

$(+)(+) = +$ | $(+)(-) = -$ | $(+)(+) = +$

$f'' > 0$ | $f'' < 0$ | $f'' > 0$

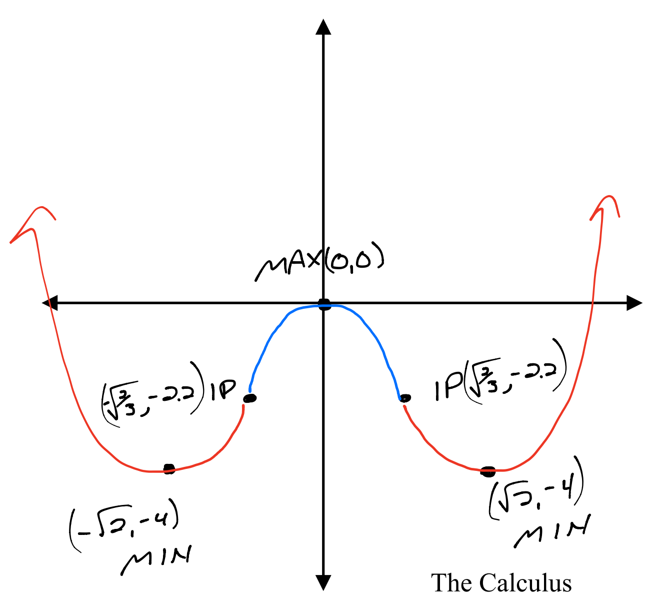
-100 $f''(-\sqrt{\frac{2}{3}}) = 0$ 0 $f''(\sqrt{\frac{2}{3}}) = 0$ 100

CCU | IP | CCD | IP | CCU

$(-\sqrt{\frac{2}{3}}, -2.2)$ $(-\sqrt{\frac{2}{3}}, -2.2)$

y-int

$f(0) = 0$



Graphing & Basic Optimization
Review Chapter 5

Graph the following function by hand by making a first derivative sign diagram and then a second derivative sign diagram.

RATIONAL

#2) $f(x) = \frac{10x^2}{x^2-9}$

$f(x) = \frac{10x^2}{(x-3)(x+3)}$

No zeros.

VA

$\boxed{\begin{array}{l|l} (x-3)(x+3)=0 & \\ x-3=0 & x+3=0 \\ x=3 & x=-3 \end{array}}$

HA

$\boxed{\begin{array}{l} n?d \\ 2=2, \text{ so } y=10 \end{array}}$

NO SA

y-int

$\boxed{f(0)=0}$

CV

$f'(x) = \frac{(10x^2)'(x^2-9) - 10x^2(x^2-9)'}{(x^2-9)^2}$

$= \frac{20x(x^2-9) - 10x^2(2x)}{(x^2-9)^2}$

$= \frac{20x^3 - 180x - 20x^3}{(x^2-9)^2}$

$= \frac{-180x}{(x^2-9)^2}$

ZON

$0 = -180x$
$0 = x$
$CV: x=0$

ZOD

$0 = (x^2-9)^2$
$0 = x^2-9$
$9 = x^2$
$\pm 3 = x$
$CV: x=-3, 3$

CP

$f(-3) = und$ VA
$f(0) = 0$
$f(3) = und$ VA

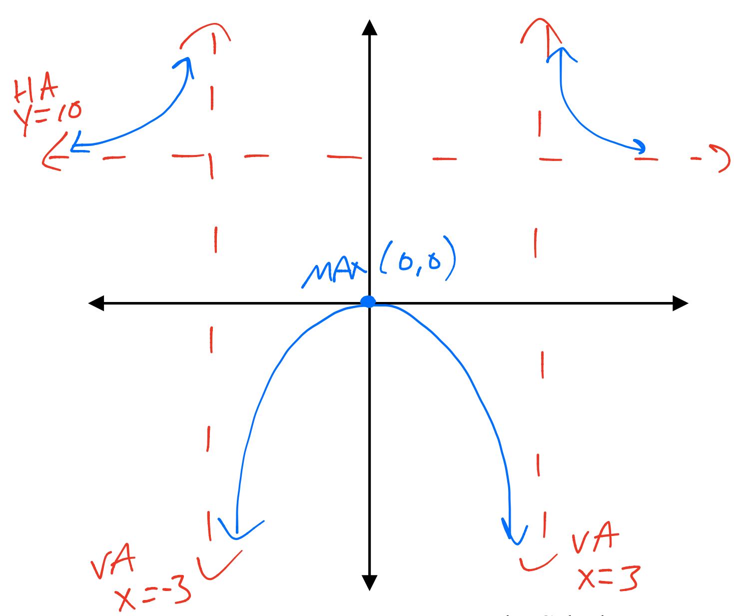
$CP: (0,0)$

$f'(x) = \frac{-180x}{(x^2-9)^2}$

$\frac{(-)(-)}{+} = +$	$\frac{(-)(-)}{+} = +$	$\frac{(-)(+)}{+} = -$	$\frac{(-)(+)}{+} = -$
$f'>0$	$f'>0$	$f'<0$	$f'<0$

-4 $f'(-3) = und$ -1 $f'(0)=0$ 1 $f'(3)=und$ 4

MAX
(0,0)



Graph the following function by hand by making a first derivative sign diagram and then a second derivative sign diagram.

#3) $f(x) = x^3 + 3x^2 + 3x + 6$

CV

$f'(x) = 3x^2 + 6x + 3$

$0 = 3(x^2 + 2x + 1)$

$0 = 3(x+1)^2$

ZON

$(x+1)^2 = 0$

$x + 1 = 0$

$x = -1$

CV: $x = -1$

CP

$f(-1) = 5$

$(-1, 5)$

$f'(x) = 3(x+1)^2$

$f' > 0$ | $f' > 0$

$f'(-1) = 0$

Neither

$(-1, 5)$

Y-int

$f(0) = 6$

CV

$f''(x) = 6x + 6$

$0 = 6(x+1)$

ZON

$x + 1 = 0$

$x = -1$

CV: $x = -1$

CP

$f(-1) = 5$

CP: $(-1, 5)$

$f''(x) = 6(x+1)$

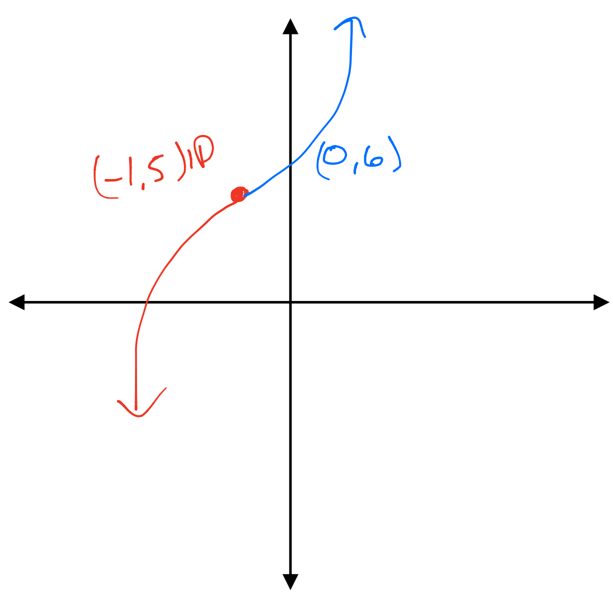
$(+)(-) = -$ | $(+)(+) = +$

$f'' < 0$ | $f'' > 0$

$f''(-1) = 0$

CCD   IP   CCU

$(-1, 5)$



Graph the following function by hand by making a first derivative sign diagram and then a second derivative sign diagram.

#4) $f(x) = x^3(x-4)$

$f(x) = x^4 - 4x^3$

CV

$f'(x) = 4x^3 - 12x^2$

$0 = 4x^2(x-3)$

ZON

$0 = 4x^2$, $0 = x^2$, $0 = x$ | $0 = x-3$, $3 = x$

CV: $x = 0, 3$

CP

$f(0) = 0$

$f(3) = -27$

CP: $(0,0)$, $(3,-27)$

CV

$f''(x) = 12x^2 - 24x$

$0 = 12x(x-2)$

ZON

$12x = 0$, $0 = x$ | $0 = x-2$, $2 = x$

CV: $x = 0, 2$

CP

$f(0) = 0$

$f(2) = -16$

CP $(0,0)$, $(2,-16)$

$f'(x) = 4x^2(x-3)$

$(+)(-) = -$	$(+)(-) = -$	$(+)(+) = +$
$f' < 0$	$f' < 0$	$f' > 0$

-1 ; $f'(0) = 0$; 1 ; $f'(3) = 0$; 4

$\searrow$ $(0,0)$ Neither $\searrow$ $(3,-27)$ MIN $\nearrow$

$f''(x) = 12x(x-2)$

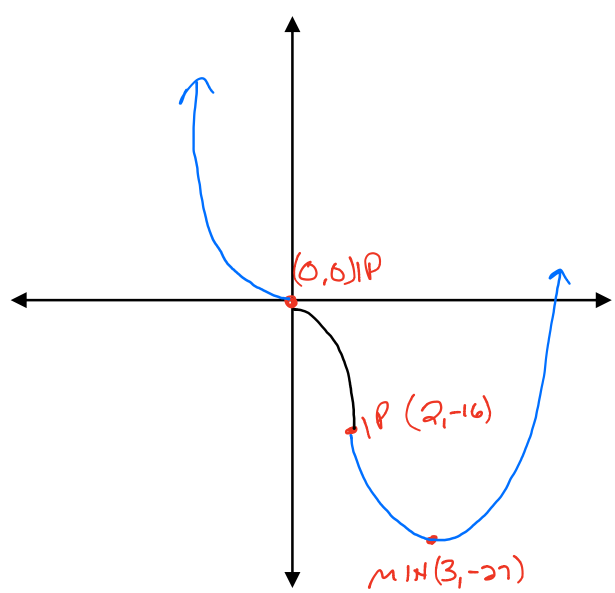
$(-)(-) = +$	$(+)(-) = -$	$(+)(+) = +$
$f'' > 0$	$f'' < 0$	$f'' > 0$

-1 ; $f''(0) = $ und ; 1 ; $f''(2) = 0$; 3

CCU ; IP $(0,0)$; CCD ; IP $(2,-16)$; CCU

Y-int

$f(0) = 0$



Find the absolute extreme points of each function on the given interval.

#5) $f(x) = x^3 - 6x^2 + 22$ on [-2, 2]

$f'(x) = 3x^2 - 12x$

$0 = 3x^2 - 12x$

$0 = 3x(x-4)$

$0 = 3x$ | $0 = x - 4$

$0 = x$ | $4 = x$

CV: $x = 0, \cancel{4}$ $4 \notin [-2, 2]$

EV: $x = -2, 2$

$f(0) = 22$ MAX

$f(-2) = -10$ MIN

$f(2) = 6$

CP: (0, 22)

EP: (-2, -10), (2, 6)

Absolute Max: (0, 22)

Absolute Min: (-2, -10)

#6) $f(x) = x^4 - 4x^3 + 4x^2$ on [0, 3]

$f'(x) = 4x^3 - 12x^2 + 8x$

$0 = 4x(x^2 - 3x + 2)$

$0 = 4x(x-2)(x-1)$

$0 = 4x$ | $0 = x - 2$ | $0 = x - 1$

$0 = x$ | $2 = x$ | $1 = x$

CV: $x = 0, 1, 2$

EV: $x = 0, 3$

$f(0) = 0$ MIN

$f(1) = 1$

$f(2) = 0$ MIN

$f(3) = 9$ MAX

CP: (0, 0), (1, 1)

EP: (0, 0), (3, 9)

Absolute Max: (3, 9)

Absolute Min: (0, 0), (2, 0)

#7) The fuel economy of an average compact car in Gnaden is $E(x) = -0.015x^2 + 1.14x + 8.3$, where x is the driving speed (in miles per hour $20 \le x \le 60$). At what speed is fuel economy the greatest? What is the greatest mpg?

$E'(x) = -0.030x + 1.14$

$0 = -0.030x + 1.14$

$0.030x = 1.14$

CV: $x = 38$

EV: $x = 20, 60$

$E(38) = 29.86$ MAX

$E(20) = 25.1$

$E(60) = 22.7$

Sentence Answer

At 38 mph the car is most efficient at 29.86 mpg.

#8) Two Wheel Deals finds that it costs \$70 to manufacture each bicycle, and fixed costs are \$100 per day. The price function is $p(x) = 270 - 10x$, where p is the price (in collars) at which exactly x bicycles will be sold. Find the quantity Two Wheel Deals should produce and the price it should charge to maximize profit. Also find the maximum profit.

$C(x) = 70x + 100$

$R(x) = p(x)\,qty$

$= (270 - 10x)x$

$R(x) = 270x - 10x^2$

$P(x) = R(x) - C(x)$

$= (270x - 10x^2) - (70x + 100)$

$P(x) = -10x^2 + 200x - 100$

$P'(x) = -20x + 200$

$0 = -20x + 200$

$20x = 20$

$x = 10$

$P''(x) = -2$

$P''(10) =$ neg, CCD, MAX

$p(10) = 270 - 10(10)$

$= 270 - 100$

$p(10) = 170$

$P(10) = -10(10)^2 + 200(10) - 100$

$= -10(100) + 2000 \quad 100$

$= -1000 + 2000 \quad 100$

$= 1000 - 100$

$P(10) = 900$

Quantity that should be produced = 10

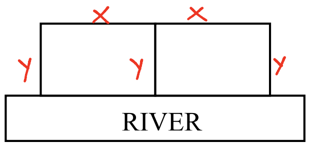
Price to be sold = \$170

Maximum Profit = \$900

Sentence Answer

Two Wheel Deals should set their price at \$170 per bike and sell 10 bikes. This will max their profit at \$900.

#9) A farmer wants to make two identical rectangular enclosures along a straight river, as in the diagram shown below. If he has 600 yards of fence, and if the sides along the river need no fence, what should be the dimensions of each enclosure if the total area is to be maximized?



x = length // to river of each enclosure
y = length ⊥ to river
A = Area
P = Perimeter

$A_{Total} = 2xy$

$A = 2x\left(200 - \frac{2}{3}x\right)$

$A = 400x - \frac{4}{3}x^2$

$P_{Fence} = 2x + 3y$

$600 = 2x + 3y$

$600 - 2x = 3y$

$200 - \frac{2}{3}x = y$

$A' = 400 - \frac{8}{3}x$

$0 = 400 - \frac{8}{3}x$

$\frac{8}{3}x = 400$

$x = \frac{\cancel{400}^{50}}{\cancel{8}} \cdot 3$

$x = 150$

$200 - \frac{2}{3}(150) = y$

$200 - 100 = y$

$100 = y$

$A''(x) = -\frac{8}{3}$

$A''(150) =$ neg, CCD, MAX

$A_{(150,\,100)} = 2(150)(100)$

$A_{(150,100)} = 30{,}000 \text{ ft}^2$

Side parallel to the river of one enclosure = 150 yards

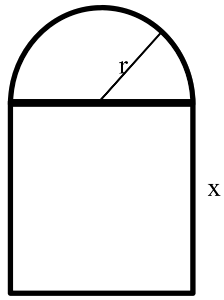
Side perpendicular to the river of one enclosure = 100 yards

Total Maximized Area = 30,000 yards^2

Sentence Answer

To max the total area to 30,000 yd^2, the side parallel to the river (of each enclosure) should be 150 yards and the sides ⊥ to river should be 100 yds.

#10) If the perimeter of the window below is to be 18 feet, find the dimensions (x and r) that maximize the area of the window. (hint: The perimeter is 3 sides of a rectangle and half a circle.)



A = Area
p = perimeter

$$A_{total} = \frac{1}{2}A_{\odot} + A_{Rec}$$
$$= \frac{1}{2}\pi r^2 + 2r(x)$$
$$A = \frac{1}{2}\pi r^2 + 2rx$$
$$A = \frac{1}{2}\pi r^2 + 2r\left(9 - r - \frac{\pi}{2}r\right)$$
$$A = \frac{1}{2}\pi r^2 + 18r - 2r^2 - \pi r^2$$

$$P = 2x + 2r + \pi r$$
$$18 = 2x + 2r + \pi r$$
$$18 - 2r - \pi r = 2x$$
$$9 - r - \frac{\pi}{2}r = x$$

$$9 - (2.5) - \frac{\pi}{2}(2.5) = x$$
$$7.5 - 1.25\pi = x$$
$$3.6 \approx x$$

$$A' = \pi r + 18 - 4r - 2\pi r$$
$$0 = 18 + \pi r - 4r - 2\pi r$$
$$-18 = r(\pi - 4 - 2\pi)$$
$$\frac{-18}{\pi - 4 - 2\pi} = r$$
$$2.5 \approx r$$

$$A = \frac{1}{2}\pi r^2 + 2rx$$
$$A(2.5, 3.6) = \frac{1}{2}\pi(2.5)^2 + 2(2.5)(3.6)$$
$$= 3.125\pi + 18$$
$$A(2.5, 3.6) \approx 27.8 \text{ ft}^2$$

$$A'' = \pi - 4 - 2\pi$$
$$A''(2.5) = \text{neg. CCD, MAX}$$

x = 2.5 feet

r = 3.6 feet

Total Maximized Area = 27.8 ft²

Sentence Answer: To max area of window to 27.8 ft², r and x should be 2.5 ft and 3.6ft, respectively.